ESP Math 182

Worksheet 7

Spring 2019

7 February 2019

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1. **Warm up:** Using the arc length formula, prove that the circumference of a circle of radius r is $2\pi r$.

2. Recall the product rule and the fact that $\displaystyle\int f'(x)\,dx = f(x)$ (we omit the constant for now).

   (a) Using these two rules, prove the integration by parts formula:

   $$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx.$$

   (b) Prove an alternative version of the integration by parts formula:

   $$\int f(x)g(x)\,dx = f(x)\int g(x)\,dx - \int f'(x)\int g(x)\,dx\,dx.$$

3. Find $f(x)$ if you know that $\displaystyle\int f(x)e^x\,dx = f(x)e^x - \int \frac{e^x}{x}\,dx.$

4. Find $\displaystyle\int (\ln(x))^k\,dx$ for any positive integer k.
   Hint: Find a pattern by computing the integral for small k.

5. Evaluate the following integrals. Be careful in choosing which method to use.

(a) $\int x\ln(x)\,dx$

(b) $\int x\cos(4x)\,dx$

(c) $\int e^{4x}\cos(3x)\,dx$

(d) $\int x^2\sin(x)\,dx$

(e) $\int x\sin(3x+4)\,dx$

(f) $\int \sin(3x)\cos(5x)\,dx$

(g) $\int \frac{x^2-\sqrt{x}}{2x}\,dx$

(h) $\int (\ln(x))^2\,dx$

(i) $\int \frac{\ln(\ln(x))}{x}\,dx$

(j) $\int \tan^2(x)\,dx$

(k) $\int \sqrt{x}e^{\sqrt{x}}\,dx$

(l) $\int x\sqrt{x+2}\,dx$

(m) $\int \frac{x^3}{(x^2+5)^2}\,dx$

(n) $\int \frac{x^3e^{x^2}}{(x^2+1)^2}\,dx$