ESP Math 182 **Spring 2019**

Worksheet 6

5 February 2019

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1. **Warm up:** Find:

   (a) $A'(x)$ for $A(x) = \displaystyle\int_3^x \sin(t^3)\ dt$

   (b) $A'(\pi)$ for $A(x) = \displaystyle\int_x^2 \frac{\cos(t)}{1+t}\ dt$

   (c) $H'(1)$ for $H(x) = \displaystyle\int_{4x^2}^9 \frac{1}{t}\ dt$

2. A sharpened pencil is 0.5cm wide, 20cm long, and its tip is sharpened to an angle of $\pi/3$.

   (a) Using solids of revolution, write down the integral that gives the total volume of the sharpened pencil.

   (b) Evaluate the integral to find the volume of this pencil.

   (c) Using geometry, describe the pencil as two simpler shapes, and calculate their volume without using solids of revolution.

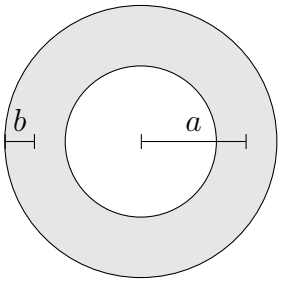
3. Find volumes of the following solids using calculus.

   (a) A ball of radius r.

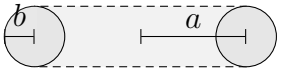
   (b) A right circular cone of radius r and height h.

   (c) A right circular cylinder of height h and radius r.

   (d) A torus of major radius a and minor radius b.

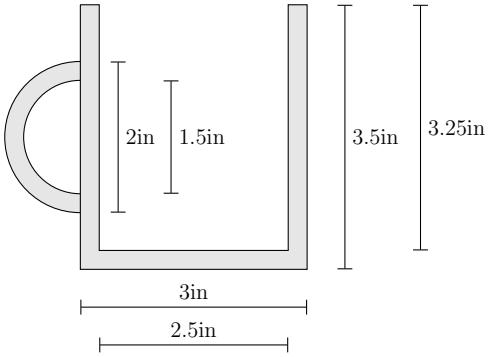


top view of a torus



side view of a torus

   (e) **Bonus:** For this part, only set up the integral, do not evaluate it: A cup with the cross section given below. You may assume the handle has the shape of a half torus.



4. Find the volume of liquid needed to fill a sphere of radius R to height h.

5. A shipping container measuring 8ft $\times$ 8.5ft $\times$ 20ft is carrying 160 circular rods of steel, of radius .5ft and height 8.5ft.

   (a) How much empty space is in the container?

   (b) If each cylindrical rod was instead a rectangular prism of dimensions 1ft $\times$ 1ft $\times$ 8.5ft, what would be the percentage increase in steel carrying capacity of the container?