

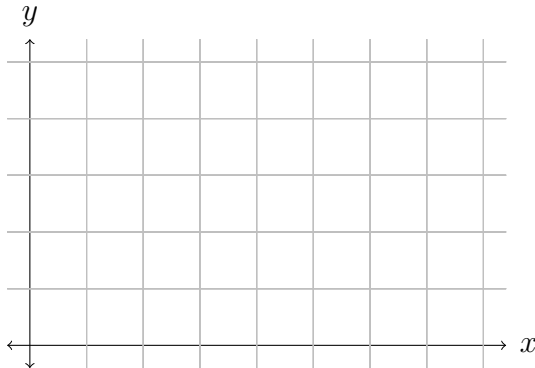
29 January 2019

1. **Warm up:** Answer the following True / False questions.

- (a) A definite integral is always ≥ 0 .
- (b) The area between curves is always ≥ 0 .
- (c) If a function is continuous, its definite integral can be calculated.
- (d) If the definite integral of a function can be calculated, the function must be continuous.
- (e) If the definite integral of a function can be calculated, the function must be differentiable.

2. Let $f(x) = \sqrt{x}$, $g(x) = x^2$ and $h(x) = 2x$ for $0 \leq x \leq 1$.

- (a) Draw the graphs of the functions on the given interval on the grid below.



- (b) Find the area of the region with all three of these functions as its boundary on this interval.

- (c) Find the volume of revolution of this area around the axis $y = -1$.

3. (a) Explain, in your own words, the volume-by-slicing method of integration.
- (b) Let S be the region bounded by $y = \frac{1}{x+1}$ and $y = 1 - \frac{x}{3}$, and let V be the solid of revolution of S about the x -axis. Use the slicing method to determine V .
- (c) Let T be the region bounded by $y = \frac{1}{x} - 1$ and $y = 3 - 3x$, and let W be the solid of revolution of T about the x -axis. Use the slicing method to determine W .
- (d) Compare your answers to parts (b) and (c) above. What is going on?
4. The two circles $x^2 + y^2 = 2$ and $x^2 + (y + 1)^2 = 1$ bound three areas in the plane. Set up, but do not evaluate the integrals expressing each of the three areas.
5. **Bonus:** Find all continuous functions $f(x)$ satisfying $\int_0^x f(t) dt = (f(x))^2 + C$.