

24 January 2019

1. Find what is wrong with this work. Can you complete it correctly?

$$\int \cos(x) \sin(x) dx$$

Let $u = \cos(x)$.

Then $du = -\sin(x)dx$.

So the integral is $-\int \cos(x) du$.

This simplifies to $-\sin(x) + C$.

2. Solve these problems by integration by substitution.

(a) $\int \frac{x}{\sqrt{x^2+9}} dx$

(e) $\int \frac{2x-1}{x^2-x} dx$

(b) $\int x^2 \sin(x^3) dx$

(f) $\int \frac{x^2 e^{\sqrt{x^3-3}}}{\sqrt{x^3-3}} dx$

(c) $\int \sin^5(x) \cos(x) dx$

(g) $\int \frac{x}{1+x} dx$

(d) $\int (x^7+2)(x^8+16x-5)^4 dx$

(h) $\int \frac{x^8}{x^3+4} dx$

3. (a) Use substitution to show that for f an even function,

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx.$$

- (b) Similarly, show that for g an odd function,

$$\int_{-a}^a g(x) dx = 0.$$

4. Evaluate the following strange-looking integrals.

$$(a) \sum_{k=1}^{20} \left(\int x^k - x^{k+1} dx \right)$$

$$(c) \int_0^9 \sqrt{4 - \sqrt{4 - \sqrt{x}}} dx$$

$$(b) \int_0^1 \left(\sum_{\ell=1}^{30} \log(x^{3\ell}) \right) dx$$

$$(d) \int \frac{1}{1 + \frac{1}{1 + \frac{1}{1+x}}} dx$$

5. Find the area between the curves $y = x^2$, $y = a|x|$, and $y = a^2$, where $a > 0$.

6. **Bonus:** Suppose that f has an inverse function f^{-1} (so $f^{-1}(f(x)) = x$ and $f(f^{-1}(y)) = y$ for all x, y). Show that

$$\int_a^b f(x) dx + \int_{f(a)}^{f(b)} f^{-1}(x) dx = bf(b) - af(a).$$

Hint: First show that $\int_{f(a)}^{f(b)} f^{-1}(x) dx = \int_a^b yf'(y) dy$ by using the substitution $x = f(y)$.