

22 January 2019

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1. Take the derivative with respect to  $x$ , and the definite integral from 0 to  $t$  with respect to  $x$ , of the following functions.

(a)  $x$

(d)  $e^x$

(b)  $e$

(e)  $e^e$

(c)  $x^e$

(f)  $ee^{ex}$

2. Let  $f(x) = 4 - x^2$  and  $g(x) = x - 2$ .

(a) What is the definition of a *graph*?

(b) Where do the graphs of  $f$  and  $g$  meet? Do they bound a finite area?

(c) Find the area of the region bounded by the curves  $f$  and  $g$ .

3. Let  $f(x) = \arcsin(3x + 1)$ .

(a) What is the domain of  $f$ ? On what sub-interval of this domain is  $f$  differentiable?

(b) Write down the equation of the tangent line to  $f$  at the point  $x = -\frac{1}{6}$ , if it exists.

4. Let  $n \geq 1$  be an integer.

(a) Draw the graph of  $y = x^n$  on  $x \in [0, 1]$  for  $n = 1, 2, 3$ .

(b) Find the integral of  $y = x^n$  on  $x \in [0, 1]$  for any  $n \geq 1$ .

(c) What happens to your answer above when  $n \rightarrow \infty$ ? Does this make sense?

5. What is the area bounded between the curves  $3 + \sin(x)$  and  $3 - \sin(x)$  from  $x = 0$  to  $x = k\pi$ , for any integer  $k$ ?

6. Find a constant  $c$  such that  $\lim_{t \rightarrow 3} \left[ \frac{4t^2 + ct + 7c - 6}{2t^2 - 5t - 3} \right]$  exists.

7. Prove that  $e^\pi > \pi^e$ . Hint: take the natural logarithm of both sides and view the statement as two functions compared at a certain value.