**ESP Math 182**

# Worksheet 2

**Spring 2019**

17 January 2019

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1. **Warm up:** Answer the following True / False questions.

   (a) A function has a unique antiderivative.

   (b) Even functions always have odd functions as antiderivatives.

   (c) If $f(a) > 0$ for some number $a$, then $F(a) > 0$ as well, for $F$ an antiderivative of $f$.

Recall the fundamental theorems of calculus. Both assume that $f$ is continuous on $(a, b)$.

**1st FTC**: $\dfrac{d}{dx}\displaystyle\int_a^x f(t)\ dt = f(x)$ for any $x \in (a, b)$.

**2nd FTC**: $\displaystyle\int_a^b f(t)\ dt = F(b) - F(a)$ for any antiderivative $F$ of $f$.

Use these to answer the questions below.

2. (a) Show that for positive numbers $a$ and $b$, $\displaystyle\int_1^a \frac{1}{t}\ dt + \int_1^b \frac{1}{t}\ dt = \int_1^{ab} \frac{1}{t}\ dt$.

   (b) Show that if $\displaystyle\int_0^x f(t)\ dt = x \cdot f(x)$, then $f$ is a constant function.

   (c) Let $g(x) = \displaystyle\int_0^x f(t)\ dt$. Using $g$, compute $\dfrac{d}{dx}\displaystyle\int_{\varphi(x)}^{\psi(x)} f(t)\ dt$ for some continuous functions $\varphi$, $\psi$. Hint: Split up the integral into two parts.

3. (a) Describe, in your own words, what is an even function and what is an odd function.

(b) Do functions that are neither even nor odd exist? If no, why? If yes, give an example.

(c) Are the two expressions below the same or not? Why?

$$\int_{-1}^{1} \frac{1}{x^2}\,dx \qquad\qquad 2\int_{0}^{1} \frac{1}{x^2}\,dx$$

4. Suppose $f$ is a continuous, $2\pi$-periodic function with $\int_0^{4\pi} f(t)\,dt = 7$. For any integer $k$, what is $\int_{k\pi}^{k\pi+2\pi} f(t)\,dt$?

5. **Bonus:** Find a function $f(t)$ and a number $a$ such that $6 + \int_a^x \frac{f(t)}{t^2}\,dt = 2\sqrt{x}$.