

1 May 2018

1. *Integral methods:* Evaluate the following integrals. Show all your work.

(a) 
$$\int \frac{x^2 e^{\sqrt{x^3-3}}}{\sqrt{x^3-3}} dx$$

(c) 
$$\int_5^7 \frac{x+1}{9x^2+4} dx$$

(b) 
$$\int x^2 \sin(2x-5) dx$$

(d) 
$$\int_e^3 \frac{x^2+x-20}{x^3-4x^2+4x} dx$$

2. *Area between curves:* Find the integral that represents the area above the curve  $y = (x-3)^2 - 12$  and below both of the curves  $y = (x-2)^3 + 5$  and  $y = 7-x$ . Do not evaluate the integral.

3. *Volumes of revolution:* Calculate the following volumes using the disk method.

(a) The area bounded by  $y = \ln(x)$ ,  $y = 4 - \ln(x)$ ,  $x = 2$ , and  $x = 4$  revolved around the  $x$ -axis.

(b) The area in the second quadrant bounded by  $x = -y^2$  and  $y = x^2$  revolved around the axis  $y = -3$ .

(c) The volume of revolution of  $y = x(x-1)(x-2)$  revolved around the  $x$ -axis between  $x = 0$  and  $x = 3$ .

4. *Sequences:* For each of the following sequences, determine if it converges or diverges. If it converges find the limit.

(a) 
$$x_n = \frac{n}{n+1}$$

(b) 
$$x_n = \frac{n \cos(n\pi)}{2n+1}$$

(c) 
$$x_n = \frac{\sin(n)}{n}$$

5. *Series:* Find the intervals of convergence of the following series. Indicate which tests you have used.

(a) 
$$\sum_{n=2}^{\infty} \frac{(x-2)^n}{(n \ln(n))^2}$$

(b) 
$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{15^n n}$$

6. *Parametric equations:*

(a) Describe the linear system

$$4x + 5y - 2z = 7,$$

$$x - y + 10z = 1$$

as a parametric equation in the variable  $t$ .

(b) For the parametric curve  $(x, y) = (5t - 2, 8 - 3t)$ , find  $\frac{dy}{dx}$  and the values of  $t$  for which the graph is in the first quadrant.

7. *Matrices:* Find the determinant, eigenvalues, and eigenvectors of the matrix  $\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$ .