ESP Math 182 **Spring 2018**

Worksheet 28

26 April 2018

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1. **Warm up 1:** Find the determinants of the following matrices.

   (a) $\begin{bmatrix} 5 & 3/4 \\ -2 & 7/3 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} -11 & 8 \\ 0 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} -2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 1/3 & 1 \end{bmatrix}$

2. **Warm up 2:** For each function f, find values a such that $f(a) = a$.

   (a) $f(x) = e^x - 1$ (b) $f(x, y) = (x, 2y - 2)$ (c) $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & 1/4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 5 \\ -2 \end{bmatrix}$

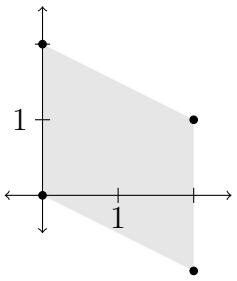
Recall that an *eigenvalue* of a matrix A is a non-zero number λ such that $A\vec{x} = \lambda\vec{x}$, for some vector $\vec{x}$, called the *eigenvector* of λ. Be careful - not all matrices have eigenvalues or eigenvectors!

3. Find the eigenvalues and associated eigenvectors and of the following linear maps.

   (a) $\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$

4. For the first three matrices above, draw where the vectors $(0, 0)$, $(1, 0)$, $(0, 1)$, $(1, 1)$ get taken to and color in the shape (called a *parallelogram*) they bound. For example:

   $T = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$ $T(0,1) = (0,2)$ $T(1,1) = (2,1)$ $T(0,0) = (0,0)$ $T(1,0) = (2,-1)$

5. Find the areas of the three shapes in the previous question. Compare them with the determinants of the corresponding linear maps.

6. Consider a function $\mathbf{R}^2 \to \mathbf{R}$ defined by $\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x & 1 \\ 1 & y \end{bmatrix}\right)$.

   (a) Take the derivative with respect to x, then with respect to y.

   (b) For what vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ is the value of the function equal to $ad - bc$?

   (c) Evaluate the derivatives at these values.