ESP Math 182 **Spring 2018**

Worksheet 27

24 April 2018

(a.) Take 5 minutes **right now** to fill out course evaluations (at least for this class). Suggestions:

- "Algebra has no place in the Math 181 curriculum and should be removed."
- "This is a calculus course, not an algebra course, so why is algebra in the syllabus?"
- "Jānis is a great instructor. He did an excellent job dealing with the algebra nonsense."

(b.) Your final Math 181 exam is Thursday, May 10, 1:00 pm - 3:00 pm. Do not miss it.

(c.) It's time to sign up for summer courses! The following are offered this year:

4-week courses (4/21 - 6/15):	Math 110, 125, 180, 220, Stat 101
8-week courses (6/18 - 8/10) :	Math 110, 125, 121, 165, 180, 181, 210, 220, 310, 417, MCS 260, 401, 471 Stat 101, 381, 401

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1. **Warm up 1:** Find coefficient vectors $\vec{x}$ that make the following equalities true.

   (a) $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$

   (b) $\begin{bmatrix} -1 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

2. **Warm up 2:** Turn the linear systems, in the given vector space, into augmented matrices.

   (a) $$\begin{aligned} x_1 - 3x_2 &= 5 \quad \text{in } \mathbf{R}^3 \\ 9 - x_2 - x_3 &= x_1 + 2 \end{aligned}$$

   (b) $$\begin{aligned} 2 + 2x_1 &= x_4 \quad \text{in } \mathbf{R}^4 \\ 1 - x_2 + 5x_1 &= 7 \\ -2 + \pi x_4 &= 0 \end{aligned}$$

3. By elementary row operations, bring the following matrix to a reduced echelon matrix (that is, make it look like the matrix on the right). Show the row operations that you carry out.

$$\begin{bmatrix} 1 & -2 & 1 & 2 & 1 \\ 1 & 1 & -1 & 1 & 2 \\ 1 & 7 & -5 & -1 & 3 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} 1 & 0 & 0 & * & * \\ 0 & 1 & 0 & * & * \\ 0 & 0 & 1 & * & * \end{bmatrix}$$

Recall that an $m \times n$ *matrix* A is a collection of mn elements, represented by A_{ij} for $1 \leqslant i \leqslant m$ and $1 \leqslant j \leqslant n$.

- The *sum* of two $m \times n$ matrices A, B is an $m \times n$ matrix $C_{ij} = A_{ij} + B_{ij}$
- The *product* of an $m \times n$ matrix A and an $n \times r$ matrix B is a $m \times r$ matrix $C_{ij} = \sum_{k=1}^{m} A_{ik} B_{kj}$
- The *transpose* of an $m \times n$ matrix A is an $n \times m$ matrix $(A^T)_{ij} = A_{ji}$
- The *inverse* of an $n \times n$ matrix A is an $n \times n$ matrix A^{-1} such that $AA^{-1} = I_n$
- To a matrix A we may apply *elementary row operations* to its rows R_k:
  - $R_k \to cR_k$, for $c \neq 0$
  - $R_k \to R_\ell$ and $R_\ell \to R_k$
  - $cR_k + R_\ell \to R_\ell$
- The *reduced echelon form* of an $m \times n$ matrix A is the $m \times n$ matrix $R = [I_m|B]$, for B an $m \times (n-m)$ matrix, and R obtained from A by elementary row operations and column swapping. We assume $n > m$.

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4. This question will explore properties of the space of all matrices.

   (a) Find two 2×2 matrices A, B such that $AB \neq BA$. This means matrices are not *commutative*.

   (b) Find two 2×2 matrices C, D such that $CD = 0$, but $C \neq 0$ and $D \neq 0$. This means matrices are not an *integral domain*.

   (c) Show that for all 2×2 matrices A, B, C, we have $(AB)C = A(BC)$. This means that matrices are *associative*.

The *determinant* of a square matrix is a number that is 0 if the matrix does not have an inverse, and nonzero otherwise. The general formula is complicated, but for small matrices we have

$$\det \begin{bmatrix} a \end{bmatrix} = a, \qquad \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc, \qquad \det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg).$$

5. The *rotation matrix* $R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ rotates any 2-vectors by an angle of θ.

   (a) What is the determinant of the rotation matrix?

   (b) Calculate R^2.

   (c) Find a matrix S such that $S^2 = R$ (the square root of R). *Hint: think geometrically.*