ESP Math 182

Worksheet 24

Spring 2017

12 April 2017

A point in *rectangular* coordinates (x, y) can be written in *polar* coordinates (r, θ), and vice versa. The correspondence is given by:

$$\begin{aligned} (x,y) &\rightarrow \left(\sqrt{x^2+y^2}, \operatorname{atan2}(y,x)\right), \\ (r\cos(\theta), r\sin(\theta)) &\leftarrow (r,\theta), \end{aligned} \qquad \operatorname{atan2}(y,x) = \begin{cases} \arctan(y/x) & \text{if } x > 0, \\ \arctan(y/x) + \pi & \text{if } x < 0, y \geqslant 0, \\ \arctan(y/x) - \pi & \text{if } x < 0, y < 0, \\ \pi/2 & \text{if } x = 0, y > 0, \\ -\pi/2 & \text{if } x = 0, y < 0, \\ 0 & \text{if } x = 0, y = 0. \end{cases}$$

1. **Warm up:** Convert the coordinates on the left to polar (r, θ) and the ones on the right to rectangular (x, y).

   (a) $(0, 0)$

   (b) $(1, 0)$

   (c) $(0, 1)$

   (d) $(1, 1)$

   (e) $(55, 78.2)$

   (f) $(0, 0)$

   (g) $(1, 0)$

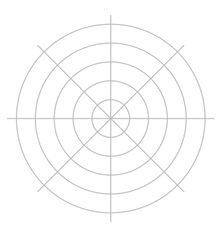
   (h) $(0, \pi)$

   (i) $(1, \pi)$

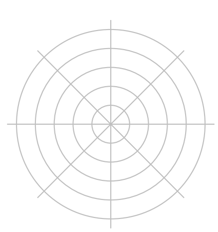
   (j) $(41/7, 22\pi/3)$

2. Draw the given polar curves $r = f(\theta)$ on the graphs below.

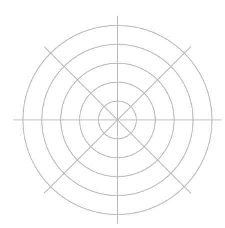
   (a) $r = 3$



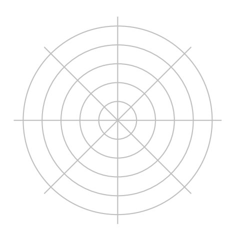
   (b) $r = \theta/\pi$



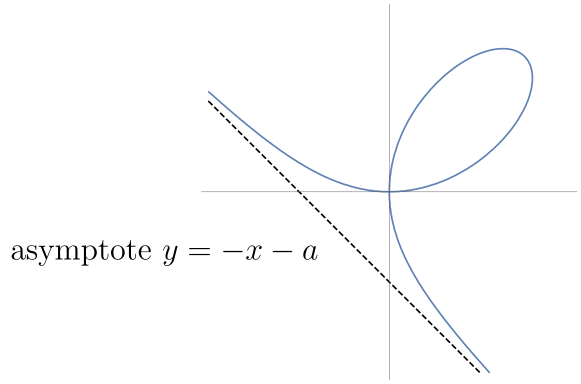
   (c) $r = 2/\cos(\theta)$



   (d) $r = 3 + \cos(3\theta)$



3. This question is about the folium of Descartes, the curve shown below. Its equation is $x^3 + y^3 = 3axy$, where $a \neq 0$ is a constant.



(a) Show that for $t \neq -1, 0$, the line $y = tx$ intersects the folium at the origin and at one other point P. Express the coordinates of P in terms of t. Use this to obtain a parametrization of the folium almost everywhere.

(b) Describe for which values of t the parametrization you found above describes the curve in quadrants I, II, and IV. Note $t = -1$ is a point of discontinuity of the parametrization.

(c) Calculate dy/dx as a function of t and find the points with horizontal or vertical tangent.

(d) Find a polar equation $r = f(\theta)$ of the folium.