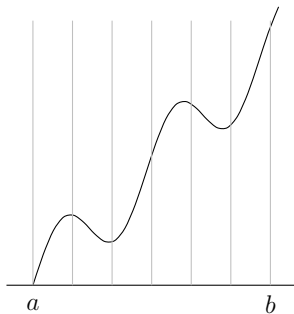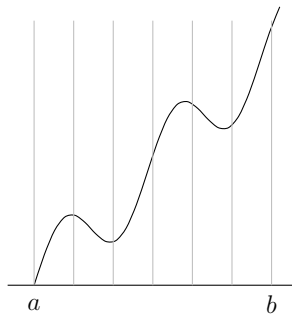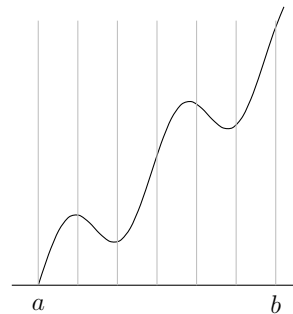# Worksheet 22

5 April 2018

1. **Warm up:** Consider a function $f$ on an interval $[a, b]$ divided up into 6 parts. Draw the shapes corresponding to the indicated integral approximation methods below.
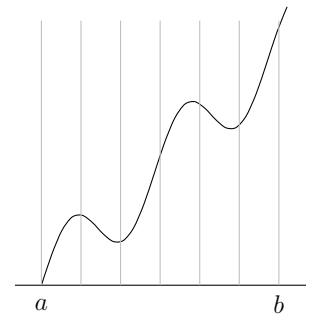


| left Riemann sum | right Riemann sum | midpoint Riemann sum | trapezoids |

Which method approximates the actual integral of $f$ from $a$ to $b$ the best? The worst?

2. Using the trapezoid sum, calculate the area underneath the curve $y = x^2$ from 0 to 1, when it is split up into $n$ intervals, then take the limit as $n \to \infty$.

3. (a) What power series represents the function $\frac{1}{1+x}$? What is the interval of convergence?

   (b) What power series represents the function $\frac{1}{1+x^2}$? What is the interval of convergence?

   (c) What power series represents the function $\arctan(x^2/2)$? What is the interval of convergence?

4. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of positive real numbers, and let $L_n = \ln(1/a_n)/\ln(n)$ for $n > 1$. Assume that $\{L_n\}_{n=1}^{\infty}$ converges, with $\lim_{n\to\infty} L_n = L$.

(a) Suppose that $L > 1$. Show that the series $\sum_{n=1}^{\infty} a_n$ converges.

(b) Suppose that $L < 1$. Show that the series $\sum_{n=1}^{\infty} a_n$ diverges.

(c) Using part (a), show that the series $\sum_{n=1}^{\infty} x^{\ln(n)}$ converges if $0 < x < 1/e$.