Worksheet 18

ESP Math 182 **Spring 2018**

15 March 2018

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1. **Warm up 1:** How many times can the following functions be repeatedly differentiated, at $x = 0$?

   (a) $f(x) = x$

   (b) $g(x) = |x|$

   (c) $h(x) = \sin(x)$

   (d) $\alpha(x) = x + |x| + \sin(x)$

   (e) $\beta(x) = \begin{cases} 0 & x \leqslant 0, \\ x^3 & x > 0. \end{cases}$

2. **Warm up 2:** Are the following expressions power series? Why or why not?

   (a) $\displaystyle\sum_{n=1}^{55} \frac{1}{n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \sum_{k=1}^{n} \frac{x^n}{n^k}$

   (e) $\displaystyle\sum_{n=0}^{\infty} x^n n^x$

   (f) $\{1, 2, 4, 8, 16, \dots\}$

   (g) $\{x, x^2, x^4, x^8, x^{16}, \dots\}$

   (h) $1 + 2 + 4 + 8 + 16 + \cdots$

   (i) $1 + x^2 + x^4 + x^8 + x^{16} + \cdots$

   (j) π

   (k) e^x

3. Consider the function $f(x) = 4x^3 - 2x^2 + 3x - 1$.

   (a) Find the 2nd order Taylor polynomial for f at $x = 0$.

   (b) Find the interval on which the Taylor approximation is no more than 0.5 away from the function.

4. For each of the following collections of points, find two different functions that go through those points.

(a) $(0,0),(1,0)$

(b) $(0,1),(2,1)$

(c) $(0,0),(\pi,0),(2\pi,0)$

(d) $(0,0),(1,1),(2,2),(3,3)$

5. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{6^n}$.

(a) Is this a geometric series? Why or why not?

(b) Find a geometric series that bounds this series above.

(c) Can you conclude that the original series converges?