ESP Math 182 | # Worksheet 16 | **Spring 2018**

8 March 2018

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1. **Warm up:** Answer the following True / False questions.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$ is a geometric series.

   (b) $\displaystyle\int_1^{100} \frac{1}{x-1}$ is an improper integral.

   (c) The sequence $1, -1, 1, -1, 1, -1, \ldots$ diverges.

2. The *integral test* states that for a non-increasing function f,

$$\sum_{n=N}^{\infty} f(n) \text{ converges} \qquad \Longleftrightarrow \qquad \int_N^{\infty} f(x)\, dx \text{ is finite.}$$

   Use the intgeral test for the questions below.

   (a) Show that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

   (b) Show that $\displaystyle\sum_{n=1000}^{\infty} \frac{1}{n}$ diverges.

   (c) Show that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$.

   (d) Calculate $\displaystyle\sum_{n=1}^{100} \frac{1}{e^n}$ and $\displaystyle\int_1^{100} \frac{1}{e^x}\, dx$.

3. Using any conergence / divergence test you have learned to determine if the following series converge or diverge. Say which test(s) you are using.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^3}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+4}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{4n^2-1}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2+\sqrt{3n}}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2+3^n}$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 2^{n+1}}{3^n}$

(i) $\displaystyle\sum_{n=1}^{\infty} \sin(1/n^2)$

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{4^n n^3}$