ESP Math 182 **Spring 2018**

Worksheet 14

1 March 2018

1. **Warm up:** Give an example of each of the following sequences. Use a different one for each!

   (a) non-increasing sequence

   (b) increasing sequence

   (c) non-decreasing sequence

   (d) decreasing sequence

   (e) constant sequence

   (f) monotonic sequence

   (g) sequence that is bounded below

   (h) sequence that is bounded above

   (i) bounded sequence

   (j) convergent sequence

   Bonus: What are the relations among the objects above? That is, which objects are specific cases of other objects? For example, "**if** constant, **then** bounded."

2. The limit of $\{a_n\}$ is L if for every $\epsilon > 0$ there exists N such that $|a_n - L| < \epsilon$ for all $n > N$. Given convergent $\{a_n\}$ and ϵ in the examples below, find L and N.

   (a) $a_n = 1/n$, $\epsilon = 1/2$

   (b) $a_n = 3/n$, $\epsilon = 2/9$

   (c) $a_n = 2^{-n} + 1$, $\epsilon = 1/1000$

   (d) $a_n = 2\cos(n\pi)/n$, $\epsilon = 1/\pi$

3. Determine if the following statements are true or false. If true, provide some justification. If false, provide a counterexample.

   (a) If $\lim_{n\to\infty} a_n = 0$ and $\lim_{n\to\infty} b_n = \infty$, then $\lim_{n\to\infty} a_n b_n = 0$.

   (b) If the sequence a_n converges, then $(-1)^n a_n$ also converges.

4. What condition on x makes $\lim_{n\to\infty} x^n = 0$? Why?

5. Notice that $0.9 = \frac{9}{10}$, $0.99 = \frac{9}{10} + \frac{9}{100}$ and so on.

   (a) Use this pattern to define a sequence $\{a_n\}$ such that $\sum_{n=1}^{\infty} a_n = 0.99999\ldots$.

   (b) Use this pattern to define a sequence $\{a_n\}$ such that $\sum_{n=1}^{\infty} a_n = 0.1234123412\ldots$.