ESP Math 182 **Spring 2018**

Worksheet 10

15 February 2018

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1. **Warm Up:** A polynomial is a function $f(x) = a_0 + a_1x + \cdots a_nx^n$ where $n \in \mathbf{Z}_{\geqslant 0}$ and $a_i \in \mathbf{R}$. Using this definition, decide which of the following functions are polynomials.

   (a) $f(x) = 0$

   (b) $g(x) = 3x + \frac{5}{2}$

   (c) $h(y) = 55y^5 + \frac{\pi^3 y^4}{e^2} + 3y^3 + 22y^2 - 2015.2$

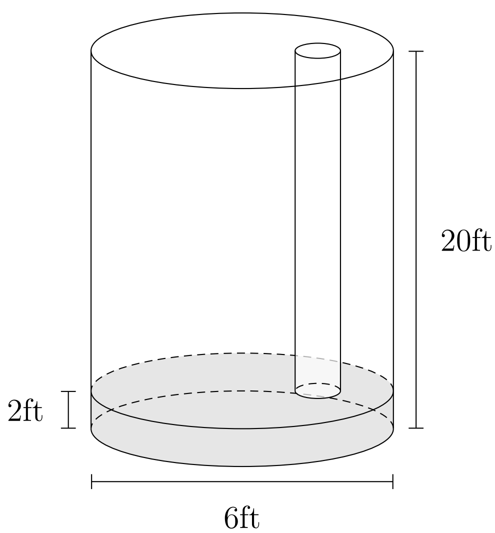
   (d) $i(z) = \frac{z^2}{5} + \frac{5}{z^2}$

   (e) $j(t) = \cos(4t^2)$

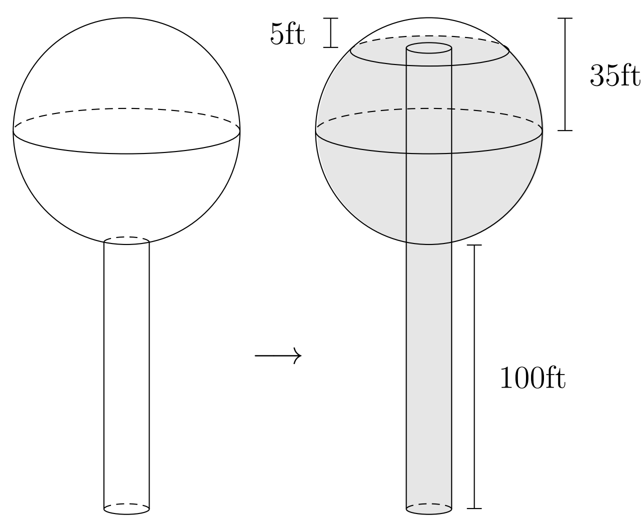
   (f) $k(q) = 99q^{99} + e^{99q}$

2. Set up integrals to solve the following water tank problems.

   (a) A cylindrical water tank has height 20 feet and diameter 6 feet, and contains water up to the 2 foot mark. A hose is dropped from the top to the surface of the water, where it floats. How much work has to be done to pump up all the water from the tank to the top of the tank?

   (b) A spherical water tank has radius 35 feet, its base is 100 feet above ground, and it is empty. An empty hose is attached to the bottom, and its end will float at the surface level as it brings water into the tank. How much work has to be done to leave five vertical feet of air in the tank?

3. Let $a \neq b$ be fixed real numbers. Prove the general formula

$$\int \frac{dx}{(x-a)(x-b)} = \frac{1}{a-b} \ln\left(\frac{x-a}{x-b}\right) + C.$$

4. Evaluate the following integrals. You will have to factor polynomials, use partial fractions, and divide polynomials by other polynomials.

(a) $\displaystyle\int \frac{dx}{x^2 - 7x + 10}$

(d) $\displaystyle\int \frac{3x^2 - 2}{x - 4}\, dx$

(b) $\displaystyle\int \frac{9 - x^2}{x - 3}\, dx$

(e) $\displaystyle\int \frac{3x + 6}{x^2(x-1)(x-3)}\, dx$

(c) $\displaystyle\int \frac{dx}{x(x^2 + x)}$

(f) **Bonus:** $\displaystyle\int \frac{5x - 1}{x^2 - 2x - 5}\, dx$

5. The *hyperbolic cosine* function $\cosh(x)$ is defined to be:

$$\cosh(x) = \frac{1}{2}(e^x + e^{-x}).$$

Find the arc length of the graph of $\cosh(x)$ on the interval $[-\ln(2), \ln(2)]$.