

13 February 2018

1. **Warm up:** List as many trigonometric identities as you can remember (without looking at your notes or the internet!)

2. Solve the following trigonometric integrals.

(a) $\int \sec(x) \tan(x) dx$

(g) $\int (\sin(x) + \cos(x))^2 dx$

(b) $\int \csc(x) dx$

(h) $\int (2 + \tan(x))^2 dx$

(c) $\int \csc^2(x) dx$

(i) $\int \frac{\cos^2(x)}{1 + \sin(x)} dx$

(d) $\int \cos^3(x) dx$

(j) $\int \sec^2(x) \sqrt{\tan(x) - 2} dx$

(e) $\int \tan(x) dx$

(k) $\int e^x \tan(e^x) dx$

(f) $\int \tan^5(x) dx$

(l) $\int \frac{\cos(2x)}{\cos(x) + \sin(x)} dx$

3. (a) What is a polynomial?

(b) Show by differentiation that if $P_n(x)$ is a polynomial of degree n which satisfies the equation $P_n(x) + P_n'(x) = x^n$, then $\int x^n e^x dx = P_n(x)e^x + C$.

4. Let $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$ for $x > 0$.

(a) Use integration by parts to show that $\Gamma(x+1) = x\Gamma(x)$ for $x > 0$.

(b) Show that $\Gamma(1) = 1$.

(c) Show that $\Gamma(n) = (n-1)!$ for all $n \in \mathbf{N}$ (the set of natural numbers).