ESP Math 182 **Spring 2018**

Worksheet 7

6 February 2018

1. **Warm up:** A strand of DNA is twisted in a double helix pattern as in the image below. If a given strand is 4mm long with a radius of $1/(10\pi)$mm and has 15 full twists, how long will the helix be when unraveled and laid flat?

2. Let A be the area between $x = 1$, $x = a > 1$, $y = 0$, and $y = 1/x$. Let V be the volume of revolution of A rotated around the x-axis. This shape is known as *Gabriel's horn*.

   (a) Find the value of V.

   (b) Find $\lim_{a\to\infty} V$.

   (c) Set up the integral (do not evaluate it) for the arclength of $y = 1/x$ on $[1, a]$.

   (d) Use the surface of revolution formula[1] to find the surface area S of V.

   (e) Find $\lim_{a\to\infty} S$.

   (f) Compare your answers to parts (b) and (e). Which is larger? Why?

[1] The surface of revolution of f on $[a, b]$ around the x-axis is $2\pi \int_a^b f(x)\sqrt{1 + (f'(x))^2}\, dx$.

3. Recall the product rule and the fact that $\int f'(x)\,dx = f(x)$ (we omit the constant for now).

   (a) Using these two rules, prove the integration by parts formula:

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx.$$

   (b) Prove an alternative version of the integration by parts formula:

$$\int f(x)g(x)\,dx = f(x)\int g(x)\,dx - \int f'(x)\int g(x)\,dx\,dx.$$

4. Solve the following volume of revolution integration problems.

   (a) Find the volume of the solid obtained by rotating $f(x) = e^x$ about the x-axis over the interval $[0, 1]$.

   (b) Find the volume left over after a sphere of radius R has a hole of radius $R/2$ drilled through the center.

   (c) Find the volume of revolution of $f(x) = |\sin(x)| + 1$ around the x-axis on the interval $[0, 3\pi]$.