

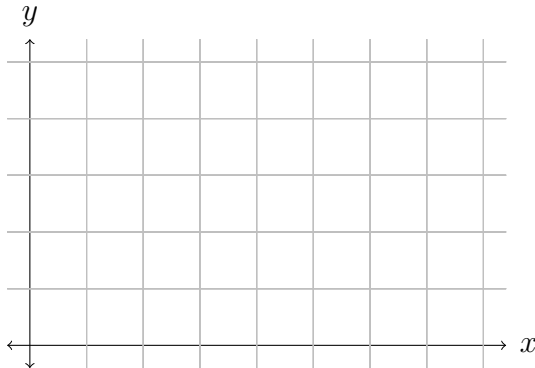
30 January 2018

1. **Warm up:** Answer the following True / False questions.

- (a) A definite integral is always ≥ 0 .
- (b) The area between curves is always ≥ 0 .
- (c) If a function is continuous, its definite integral can be calculated.
- (d) If the definite integral of a function can be calculated, the function must be continuous.
- (e) If the definite integral of a function can be calculated, the function must be differentiable.

2. Let $f(x) = \sqrt{x}$, $g(x) = x^2$ and $h(x) = 2x$ for $0 \leq x \leq 1$.

(a) Draw the graphs of the functions on the given interval on the grid below.



(b) Find the area of the region with all three of these functions as its boundary on this interval.

(c) Find the volume of revolution of this area around the axis $y = -1$.

3. (a) Explain, in your own words, the volume-by-slicing method of integration.
- (b) Let S be the region bounded by $y = \frac{1}{x+1}$ and $y = 1 - \frac{x}{3}$, and let V be the solid of revolution of S about the x -axis. Use the slicing method to determine V .
- (c) Let T be the region bounded by $y = \frac{1}{x} - 1$ and $y = 3 - 3x$, and let W be the solid of revolution of T about the x -axis. Use the slicing method to determine W .
- (d) Compare your answers to parts (b) and (c) above. What is going on?
4. The two circles $x^2 + y^2 = 2$ and $x^2 + (y + 1)^2 = 1$ bound three areas in the plane. Set up, but do not evaluate the integrals expressing each of the three areas.
5. **Bonus:** Find all continuous functions $f(x)$ satisfying $\int_0^x f(t) dt = (f(x))^2 + C$.