ESP Math 182 **Spring 2018**

Worksheet 4

25 January 2018

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1. Find what is wrong with this work. Can you complete it correctly?

   $\int \cos(x)\sin(x)\ dx$

   Let $u = \cos(x)$.

   Then $du = -\sin(x)dx$.

   So the integral is $-\int \cos(x)\ du$.

   This simplifies to $-\sin(x) + C$.

2. Solve these problems by integration by substitution.

   (a) $\displaystyle\int \frac{x}{\sqrt{x^2+9}}\ dx$

   (b) $\displaystyle\int x^2 \sin(x^3)\ dx$

   (c) $\displaystyle\int \sin^5(x)\cos(x)\ dx$

   (d) $\displaystyle\int (x^7+2)(x^8+16x-5)^4\ dx$

   (e) $\displaystyle\int \frac{2x-1}{x^2-x}\ dx$

   (f) $\displaystyle\int \frac{x^2 e^{\sqrt{x^3-3}}}{\sqrt{x^3-3}}\ dx$

   (g) $\displaystyle\int \frac{x}{1+x}\ dx$

   (h) $\displaystyle\int \frac{x^8}{x^3+4}\ dx$

3. (a) Use substitution to show that for f an even function,

   $$\int_{-a}^{a} f(x)\ dx = 2\int_{0}^{a} f(x)\ dx.$$

   (b) Similarly, show that for g an odd function,

   $$\int_{-a}^{a} g(x)\ dx = 0.$$

4. Suppose that f has an inverse function f^{-1} (so $f^{-1}(f(x)) = x$ and $f(f^{-1}(y)) = y$ for all x, y). Show that

$$\int_a^b f(x)\ dx + \int_{f(a)}^{f(b)} f^{-1}(x)\ dx = bf(b) - af(a).$$

Hint: First show that $\displaystyle\int_{f(a)}^{f(b)} f^{-1}(x)\ dx = \int_a^b yf'(y)\ dy.$

5. Evaluate the following strange-looking integrals.

(a) $\displaystyle\sum_{k=1}^{20} \left(\int x^k - x^{k+1}\ dx \right)$

(b) $\displaystyle\int_0^1 \left(\sum_{\ell=1}^{30} \log\left(x^{3\ell}\right) \right)\ dx$

(c) $\displaystyle\int_0^9 \sqrt{4 - \sqrt{4 - \sqrt{x}}}\ dx$

(d) $\displaystyle\int \frac{1}{1 + \frac{1}{1+\frac{1}{1+x}}}\ dx$

6. Find the area between the curves $y = x^2$, $y = a|x|$, and $y = a^2$, where $a > 0$.