Worksheet 3

23 January 2018

1. Take the derivative with respect to x, and the definite integral from 0 to t with respect to x, of the following functions.

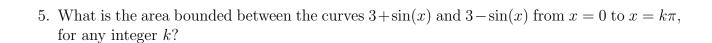   (a) x

    (b) e

    (c) x^e

    (d) e^x

    (e) e^e

    (f) ee^{ex}

2. Let $f(x) = 4 - x^2$ and $g(x) = x - 2$.

    (a) What is the definition of a *graph*?

    (b) Where do the graphs of f and g meet? Do they bound a finite area?

    (c) Find the area of the region bounded by the curves f and g.

3. Let $f(x) = \arcsin(3x + 1)$.

    (a) What is the domain of f? On what sub-interval of this domain is f differentiable?

    (b) Write down the equation of the tangent line to f at the point $x = -\frac{1}{6}$, if it exists.

4. Let $n \geqslant 1$ be an integer.

    (a) Draw the graph of $y = x^n$ on $x \in [0, 1]$ for $n = 1, 2, 3$.

    (b) Find the integral of $y = x^n$ on $x \in [0, 1]$ for any $n \geqslant 1$.

    (c) What happens to your answer above when $n \to \infty$? Does this make sense?

5. What is the area bounded between the curves $3+\sin(x)$ and $3-\sin(x)$ from $x=0$ to $x=k\pi$, for any integer k?

6. Find a constant c such that $\lim\limits_{t\to 3}\left[\dfrac{4t^2+ct+7c-6}{2t^2-5t-3}\right]$ exists.

7. Prove that $e^\pi > \pi^e$. Hint: take the natural logarithm of both sides and view the statement as two functions compared at a certain value.