

20 April 2017

1. **Warm up:** Give the definition, in your own words, of the following terms.

- (a) matrix
- (b) eigenvector
- (c) eigenvalue

2. Find the eigenvalues, eigenvectors, and determinants of the following linear maps.

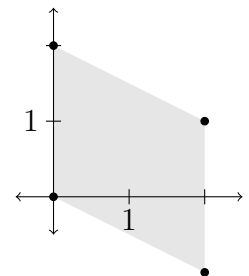
(a)  $\begin{bmatrix} 5 & 0 \\ -10 & -2 \end{bmatrix}$

(b)  $\begin{bmatrix} 7/2 & 3 \\ -1 & 10 \end{bmatrix}$

3. For the maps above, draw where the vectors  $(0, 0)$ ,  $(1, 0)$ ,  $(0, 1)$ ,  $(1, 1)$  get taken to and color in the shape (called a *parallelogram*) they bound. For example:

$$T = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$$

$$\begin{array}{ll} T(0, 1) = (0, 2) & T(1, 1) = (2, 1) \\ T(0, 0) = (0, 0) & T(1, 0) = (2, -1) \end{array}$$



4. Find the areas of the parallelograms in the previous question. Compare them with the determinants of the corresponding linear maps.

5. Let  $\vec{x} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$  be a vector in  $\mathbf{R}^2$ .

- (a) Find a  $2 \times 2$  matrix  $A$  such that  $A\vec{x} = \begin{bmatrix} 1 \\ 1/5 \end{bmatrix}$ .
- (b) Find a  $2 \times 2$  matrix  $B$  that rotates  $\vec{x}$  by 120 degrees clockwise.
- (c) Draw the vectors  $\vec{x}$ ,  $A\vec{x}$ , and  $BA\vec{x}$  on a grid. Describe, in words, what the matrix  $BA$  does to  $\vec{x}$ .