ESP Math 182

Worksheet 27

Spring 2017

18 April 2017

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1. **Warm up 1:** Fill out your student evaluations for this class - you should have gotten them by email. Here are some suggestions:

   (a) Jānis is an excellent instructor and deserves a raise.

   (b) Jānis did a superb job dealing with the absolute nonsense of the Calc 2 syllabus.

   (c) Jānis's teaching showed that regular discussion sections are worthless and all students should participate in ESP workshops.

2. **Warm up 2:** For each function f, find values a such that $f(a) = a$.

   (a) $f(x) = e^x - 1$ (b) $f(x, y) = (x, 2y - 2)$ (c) $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & 1/4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 5 \\ -2 \end{bmatrix}$

Recall that an *eigenvalue* of a linear transformation T is a vector $\vec{x}$, called an *eigenvector*, such that $T\vec{x} = \lambda\vec{x}$, for some non-zero number λ.

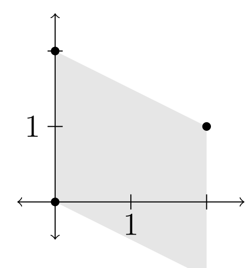
3. Find the eigenvalues and associated eigenvectors and of the following linear maps.

   (a) $\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. For the first three maps above, draw where the vectors $(0, 0)$, $(1, 0)$, $(0, 1)$, $(1, 1)$ get taken to and color in the shape (called a *parallelogram*) they bound. For example:

   $T = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$

   $T(0, 1) = (0, 2)$ $T(1, 1) = (2, 1)$
   $T(0, 0) = (0, 0)$ $T(1, 0) = (2, -1)$



5. Find the areas of the three shapes in the previous question. Compare them with the determinants of the corresponding linear maps.

6. Consider a map $\mathbf{R}^2 \to \mathbf{R}$ defined by $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] \mapsto \det\left(\left[\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right]\left[\begin{smallmatrix} x & 1 \\ 1 & y \end{smallmatrix}\right]\right)$. Take the derivative with respect to x, then with respect to y (both variables are independent of each other), and evaluate the derivatives at the determinant of $\left[\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right]$.