ESP Math 182 # Worksheet 26 **Spring 2017**

13 April 2017

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1. **Warm up:** Simplify the following expressions.

   (a) $\det \begin{bmatrix} 3 & 2 \\ -2 & 7 \end{bmatrix}$ (b) $\sum_{i=1}^{n} \begin{bmatrix} i & 2i \\ i/2 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}^8$

2. This question will have you prove *Euler's formula*.

   (a) Write down the power series for e^x, $\sin(x)$, and $\cos(x)$, all centered at 0.

   (b) Using part (a), express e^{ix} as a power series. Then separate all the terms with and without a coefficient of i.

   (c) Using parts (a) and (b), rewrite e^{ix} using $\sin(x)$ and $\cos(x)$. This is known as Euler's formula.

3. Using Euler's formula, rewrite the complex numbers on the left as $r\cos(\theta) + ir\sin(\theta)$ and the numbers on the right as $re^{i\theta}$.

   (a) $e^{i\pi/3}$ (d) $\cos(7\pi/10) + i\sin(7\pi/10)$

   (b) $\pi e^{i\pi}$ (e) $\sqrt{3}/2 + i/2$

   (c) $5e^{2i}$ (f) $7 + 9i$

4. The *rotation matrix* $R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ rotates any 2-vectors by an angle of θ.

   (a) What is the determinant of the rotation matrix?

   (b) Calculate R^2. What does this mean geometrically?

   (c) Find a matrix S such that $S^2 = R$ (the square root of R).

5. Let $a, b, c, d, f, g \in \mathbf{R}$. Complex numbers $a + ib$ can be expressed as 2-vectors $\begin{bmatrix} a \\ b \end{bmatrix}$.

   (a) Write $a + ib$ in the form $re^{i\theta}$ using Euler's formula.

   (b) For some other complex number $c + id$, express the product $(a + ib)(c + id)$ in the form $re^{i\theta}$. Use part (a) above to make your work simpler.

   (c) Call the product $f + ig$ from part (b) above. Using the rotation matrix, find a 2×2 matrix for which

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}.$$