ESP Math 182 **Spring 2017**

Worksheet 23

4 April 2017

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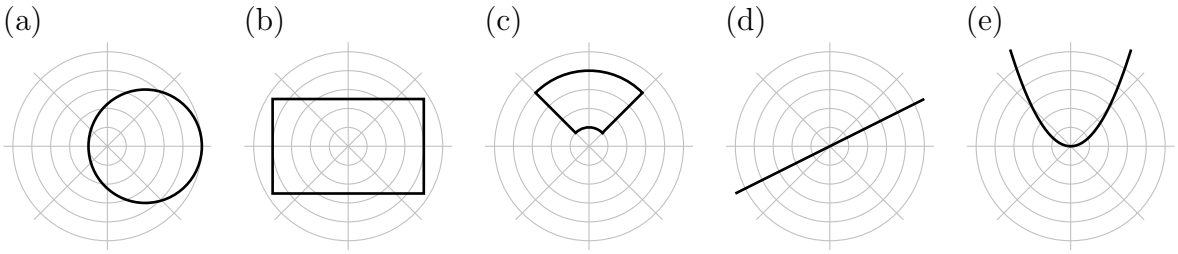
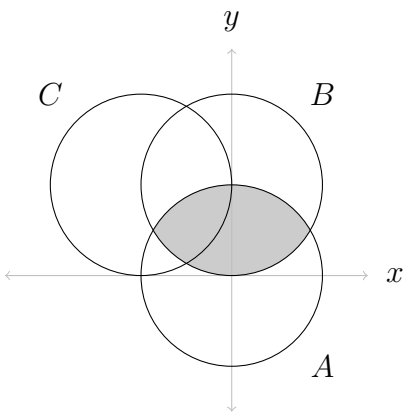
1. **Warm up:** Determine which of the shapes below could come from polar functions of r and which can not. (*Bonus: Find the polar functions that have them as graphs.*)



2. (a) Describe the three unit circles below as polar equations $r = f(\theta)$.

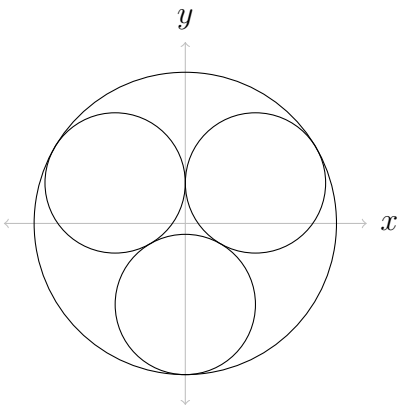


   (b) Find the area inside both A and B (the shaded area). Be careful with the bounds of your integral, make sure you know between which part of the curves you are integrating.

   (c) Find the area inside A but not inside B in the first quadrant.

   (d) Describe the area inside both A and C by an integral, but do not solve the integral.

3. Consider the three circles of the same radius inscribed inside the unit circle below. The inside circles are tangent to the unit circle at $\theta = \pi/6, 5\pi/6, 3\pi/2$, and are tangent to each other at $\theta = \pi/2, 7\pi/6, 11\pi/6$.

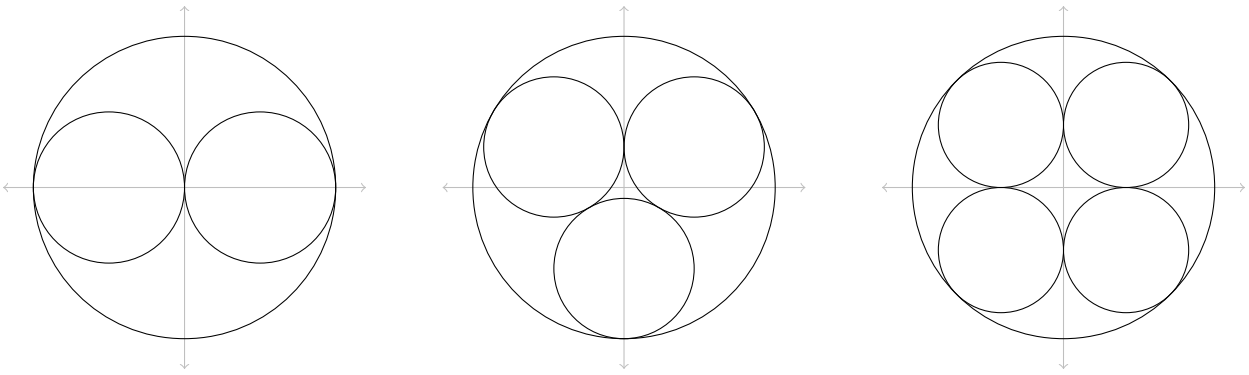


(a) Find the radius of the three inscribed circles and their centers.

(b) Using integrals, find the area of the three-sided shape bounded by the three circles at the center of the diagram above.

(c) **Bonus:** Find the radius of the largest circle that can fit between two of the inside circles and the bounding unit circle.

4. Find the areas of the two, three, and four inscribed circles (all inside unit circles) below.



Which collection of inscribed circles covers the most area?