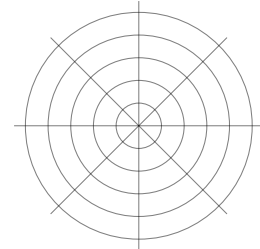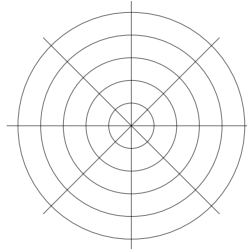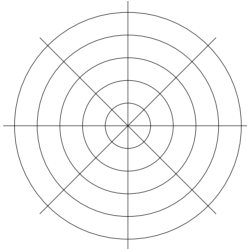# Worksheet 22

30 March 2017

1. **Warm up:** Draw the regions described by the inequalities in polar coordinates below.

   (a) $r \leqslant 3$, $\pi \leqslant \theta < 2\pi$ (b) $r < 3$, $\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{2}$ (c) $3 \leqslant r \leqslant 5$, $\frac{3\pi}{4} \leqslant \theta \leqslant \pi$

2. This question is about the folium of Descartes, the curve shown below. Its equation is $x^3 + y^3 = 3axy$, where $a \neq 0$ is a constant.

   asymptote $y = -x - a$
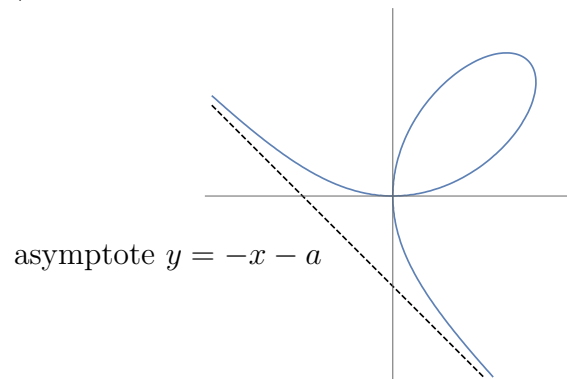
   (a) Show that for $t \neq -1, 0$, the line $y = tx$ intersects the folium at the origin and at one other point P. Express the coordinates of P in terms of t. Use this to obtain a parametrization of the folium almost everywhere.

   (b) Describe for which values of t the parametrization you found above describes the curve in quadrants I, II, and IV. Note $t = -1$ is a point of discontinuity of the parametrization.

   (c) Calculate dy/dx as a function of t and find the points with horizontal or vertical tangent.

   (d) Find a polar equation $r = f(\theta)$ of the folium.

3. Find the intervals of convergence of the following series. Remeber to check endpoints.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2^n}$

(b) $\displaystyle\sum_{n=0}^{\infty} n^n x^n$

(c) $\displaystyle\sum_{n=1}^{\infty} n(x-3)^n$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n x^n}{n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-5)^n (x-3)^n}{n^2}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n2^n}$

(g) $\displaystyle\sum_{n=12}^{\infty} e^n (x-2)^n$

(h) $\displaystyle\sum_{n=2}^{\infty} \frac{(x-2)^n}{(n\ln(n))^2}$