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1. By the theory of relativity, the length of an object moving relative to an observer changes depending upon how fast the object is traveling (relative to the observer). This phenomenon is called *length contraction / dilation*, or *Lorentz contraction*. By experimental results, the length of the observed object seems to be governed by the equation

$$L(v) = \ell \sqrt{1 - \frac{v^2}{c^2}}.$$

Here v is the speed of the object in meters per second, ℓ is the length of the object (in meters) at rest (relative to the observer), and c is the speed of light, $c \approx 3 \cdot 10^9$ meters per second.

- (a) Find the following approximations to L at $v = 0$:
- linear,
 - quadratic,
 - cubic,
 - quartic.
- (b) Guess as to what the Maclaurin series of L should be in general.
- (c) Using the quadratic approximation, estimate:
- the length of an observed car moving at 100 kilometers per hour, by a pedestrian on the sidewalk, if the car is 3 meters long at rest;
 - the length of the observed spacecraft *Helios-B*, traveling at $\approx 2.5 \cdot 10^5$ kilometers per hour, by an observer on the earth, if the spacecraft is 32 meters long at rest;
 - the speed at which a bicycle of length 2 meters (at rest) must be traveling to look 1 meter long to a bystander.

Try the above without a calculator!