ESP Math 182

Worksheet 19

Spring 2017

14 March 2017

1. **Warm up:** Let m, n be integers. Evaluate the following expressions.

   (a) $\displaystyle\int_0^\pi \sin^2(mx)\,dx$

   (b) $\displaystyle\int_0^\pi \sin(mx)\sin(nx)\,dx$

   (c) $\displaystyle\int_0^\pi \cos(mx)\cos(nx)\,dx$

2. Show that for any integer $n \geqslant 2$, the identity

$$\int \sec^n(t)dt = \frac{1}{n-1}\sec^{n-2}(t)\tan(t) + \frac{n-2}{n-1}\int \sec^{n-2}(t)dt$$

   holds, and use it to calculate $\int \sec^6(t)dt$.

3. Recognize that $\sum_{k=0}^{\infty} 2^k x^{2k+1}$ is the Taylor series for some function. Identify this function, as well as the center, radius, and interval of convergence for the series.

4. Recall the power series for $\frac{1}{1-x}$ for $|x| < 1$. Using it, find power series for the following functions, indicating where they are centered and for what values of x they work.

(a) $\dfrac{x}{x-1}$ (b) $\dfrac{1}{1+x}$ (c) $\dfrac{1}{x-2}$

5. (a) Show that $\dfrac{1}{1-x} = \dfrac{-1/3}{1+(x-4)/3}$.

(b) For $|x-4| < 3$, show that $\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty}(-1)^{n-1}\frac{(x-4)^n}{3^{n+1}}$.

(c) For $|x-4| > 3$, show that $\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty}(-1)^{n-1}\frac{3^n}{(x-4)^{n+1}}$.

6. Find the Maclaurin series and intervals of convergence for the following functions.

(a) $f(x) = \dfrac{e^{x^2}}{x} - \dfrac{1}{(1-3x)^2}$

(b) $g(x) = \dfrac{1}{3x^2+2} + \cos(3x)$