ESP Math 182

Worksheet 17

Spring 2017

7 March 2017

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1. **Warm up:** Are the following expressions power series? Why or why not?

   (a) $\displaystyle\sum_{n=1}^{55} \frac{1}{n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \sum_{k=1}^{n} \frac{x^n}{n^k}$

   (e) $\displaystyle\sum_{n=0}^{\infty} x^n n^x$

   (f) $\{1, 2, 4, 8, 16, \dots\}$

   (g) $\{x, x^2, x^4, x^8, x^{16}, \dots\}$

   (h) $1 + 2 + 4 + 8 + 16 + \cdots$

   (i) $1 + x^2 + x^4 + x^8 + x^{16} + \cdots$

   (j) π

   (k) e^x

2. Consider the function $f(x) = 4x^3 - 2x^2 + 3x - 1$.

   (a) Find the 2nd order Taylor polynomial for f at $x = 0$.

   (b) Find the interval on which the Taylor approximation is no more than 0.5 away from the function.

3. Consider the series $\sum_{n=1}^{\infty} \frac{n^2}{6^n}$.

   (a) Is this a geometric series? Why or why not?

   (b) Find a geometric series that bounds this series above.

   (c) Can you conclude that the original series converges?

4. The *root test* says that a series $\sum_{n=1}^{\infty} a_n$ converges if $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$ and diverges if > 1.

   (a) Use the root test to find the radius of convergence of $\sum_{n=1}^{\infty} \sin(1/n)x^n$.

   (b) Use the root test to find the radius of convergence of the geometric series $\sum_{n=0}^{\infty} ar^n$.

5. Find a power series with the following intervals of convergece:

   (a) $(1/2, 5/2)$

   (b) (a, b) for any pair of real numbers $a < b$