ESP Math 182 **Spring 2017**

Worksheet 16

2 March 2017

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1. **Warm up 1:** Describe, in your own words, what the following terms mean.

   (a) polynomial

   (b) Taylor polynomial

   (c) first derivative of a function f at a point p

2. **Warm up 2:** How many times can the following functions be repeatedly differentiated, at $x = 0$?

   (a) $f(x) = x$

   (b) $g(x) = |x|$

   (c) $h(x) = \sin(x)$

   (d) $\alpha(x) = x + |x| + \sin(x)$

   (e) $\beta(x) = \begin{cases} 0 & x \leqslant 0, \\ x^2 & x > 0. \end{cases}$

3. (a) For each of the following collections of points, find two different functions that go through those points.

   i. $(0, 0), (1, 0)$

   ii. $(0, 1), (2, 1)$

   iii. $(0, 0), (\pi, 0), (2\pi, 0)$

   iv. $(0, 0), (1, 1), (2, 2), (3, 3)$

   (b) Find the 1st and 2nd order Taylor polynomials for one function from each part above. What are their y-values at the x-values of the given points?

4. You are given the two inequalities

$$x - \frac{x^2}{2} < \ln(1+x) < x, \qquad \text{for } x > 0, \qquad (1)$$

$$\ln(1) + \ln(2) + \cdots + \ln(n-1) < \int_1^n \ln(x)\ dx < \ln(2) + \cdots + \ln(n), \qquad \text{for } n \geqslant 2. \qquad (2)$$

Use them to answer the following questions.

(a) Use inequality (1) to prove that $\lim_{x\to 0^+} \left[\frac{\ln(1+x)}{x}\right] = 1.$

(b) Use part (a) to prove that $\lim_{n\to\infty} \left[\left(1 + \frac{1}{n}\right)^n\right] = e.$

(c) Show that $\sum_{n=1}^{\infty} \frac{a^n n!}{n^n}$ converges for $0 < a < e$ and diverges for $a > e$.

(d) Use inequality (2) to show that $\frac{n^n}{e^{n-1}} < n! < \frac{(n+1)^{n+1}}{e^n}$.

(e) Use part (d) to determine if the series from part (c) converges if $a = e$.