ESP Math 182 **Spring 2017**

Worksheet 15

28 February 2017

---

1. **Warm up:** Answer the following true / false questions.

   (a) The sequence $\{a_n\}_{n=1}^{\infty}$ for $a_n = \frac{1}{n}$ converges.

   (b) The series $\sum_{n=1}^{\infty} \frac{1}{n}$ converges.

   (c) If a series $\sum_{n=0}^{\infty} a_n$ converges and $a_n \to c$ as $n \to \infty$, then $c = 0$.

   (d) If a sequence $\{a_n\}_{n=1}^{\infty}$ converges to 0, then $\sum_{n=0}^{\infty} a_n$ converges.

2. Use geometric series to show that:

   (a) $0.99999\ldots\ldots = 1$

   (b) $0.5555555\ldots\ldots = 5/9$

   (c) $1.36363636\ldots\ldots = 15/11$

3. Recall that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

   (a) Reindex this series so that the index starts at $n = 0$. That is, keep the series the same, but change the $\frac{1}{n}$ to something else.

   (b) Use part (a) to show that $\sum_{n=1}^{\infty} \frac{1}{n+1}$ diverges.

   (c) Use these ideas to show that, for any positive integer k, the series $\sum_{n=1}^{\infty} \frac{1}{n+k}$ diverges.

4. Using the p-test, ratio test, (limit) comparison test, and alternating series test, determine if the following series converge or diverge. Say which convergence test(s) you are using.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+4}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{4n^2-1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2+\sqrt{3n}}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2+3^n}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 2^{n+1}}{3^n}$

(h) $\displaystyle\sum_{n=1}^{\infty} (-1)^n(\pi/2 - \arctan(n))$

(i) $\displaystyle\sum_{n=1}^{\infty} \sin(1/n^2)$

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{4^n n^3}$