ESP Math 182 | # Worksheet 14 | **Spring 2017**

23 February 2017

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1. **Warm up:** Give an example of each of the following sequences. Use a different one for each!

   (a) non-increasing sequence

   (b) increasing sequence

   (c) non-decreasing sequence

   (d) decreasing sequence

   (e) constant sequence

   (f) monotonic sequence

   (g) sequence that is bounded below

   (h) sequence that is bounded above

   (i) bounded sequence

   (j) convergent sequence

   Bonus: What are the relations among the objects above? That is, which objects are specific cases of other objects? For example, "**if** constant, **then** bounded."

2. Determine if the following statements are true or false. If true, provide some justification. If false, provide a counterexample.

   (a) If $\lim_{n\to\infty} a_n = 0$ and $\lim_{n\to\infty} b_n = \infty$, then $\lim_{n\to\infty} a_n b_n = 0$.

   (b) If the sequence a_n converges, then $(-1)^n a_n$ also converges.

   (c) $\displaystyle\sum_{n=0}^{k}(a_n + b_n) = \sum_{n=0}^{k} a_n + \sum_{n=0}^{k} b_n$ for $k < \infty$

   (d) $\displaystyle\sum_{n=0}^{\infty}(a_n + b_n) = \sum_{n=0}^{\infty} a_n + \sum_{n=0}^{\infty} b_n$

   (e) $\displaystyle\sum_{n=0}^{\infty} a_n b_n = \left(\sum_{n=0}^{\infty} a_n\right)\left(\sum_{n=0}^{\infty} b_n\right)$

3. Determine if the following infinite series converge. If so, find the sum.

(a) $\dfrac{1}{10} + \dfrac{3}{20} + \dfrac{9}{40} + \dfrac{27}{80} + \dfrac{81}{160} + \cdots$

(b) $\dfrac{3}{4} + \dfrac{1}{4} + \dfrac{1}{12} + \dfrac{1}{36} + \dfrac{1}{108} + \cdots$

(c) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n 2^{2-3n}$

(d) $\displaystyle\sum_{n=0}^{\infty} (-1)^n e^{3-n} 2^{n+1} - \left(\frac{2}{3}\right)^{2n}$

(e) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \left(\frac{2}{3}\right)^{2n} + \frac{3 \cdot 8^n}{81^{n/2}}$

4. Use geometric series to show that:

(a) $0.99999\ldots\ldots = 1$

(b) $0.5555555\ldots\ldots = 5/9$

(c) $1.36363636\ldots\ldots = 15/11$