ESP Math 182

Worksheet 9

Spring 2017

7 February 2017

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1. **Warm Up:** Evaluate the following integrals. You will have to factor polynomials, use partial fractions, and divide polynomials by other polynomials.

   (a) $\displaystyle\int \frac{dx}{x^2 - 7x + 10}$

   (b) $\displaystyle\int \frac{9 - x^2}{x - 3}\, dx$

   (c) $\displaystyle\int \frac{dx}{x(x^2 + x)}$

   (d) $\displaystyle\int \frac{3x^2 - 2}{x - 4}\, dx$

   (e) $\displaystyle\int \frac{3x + 6}{x^2(x - 1)(x - 3)}\, dx$

   (f) **Bonus:** $\displaystyle\int \frac{5x - 1}{x^2 - 2x - 5}\, dx$

2. Let $a \neq b$ be fixed real numbers. Prove the general formula

$$\int \frac{dx}{(x-a)(x-b)} = \frac{1}{a-b} \ln\left(\frac{x-a}{x-b}\right) + C.$$

3. (a) What is a polynomial?

(b) Show by differentiation that if $P_n(x)$ is a polynomial of degree n which satisfies the equation $P_n(x) + P_n'(x) = x^n$, then $\int x^n e^x dx = P_n(x)e^x + C$.

4. Let $\Gamma(x) = \displaystyle\int_0^\infty e^{-t}t^{x-1}dt$ for $x > 0$.

(a) Use integration by parts to show that $\Gamma(x+1) = x\Gamma(x)$ for $x > 0$.

(b) Show that $\Gamma(1) = 1$.

(c) Show that $\Gamma(n) = (n-1)!$ for all $n \in \mathbf{N}$ (the set of natural numbers).