

2 February 2017

1. **Warm up:** Recall the product rule and the fact that $\int f'(x) dx = f(x)$.

(a) Use these facts to prove the integration by parts formula:

$$\int f(x)g'(x) dx = f(x)g(x) - \int f'(x)g(x) dx.$$

(b) Use these facts to prove another version of the integration by parts formula:

$$\int f(x)g(x) dx = f(x) \int g(x) dx - \int f'(x) \int g(x) dx dx.$$

2. Evaluate the following integrals. Be careful in choosing which method to use.

(a) $\int x \ln(x) dx$

(e) $\int \frac{x^2 - \sqrt{x}}{2x} dx$

(b) $\int e^{4x} \cos(3x) dx$

(f) $\int (\ln(x))^2 dx$

(c) $\int x^2 \sin(x) dx$

(g) $\int \tan^2(x) dx$

(d) $\int x \sin(3x + 4) dx$

(h) $\int x\sqrt{x+2} dx$

3. Using the arc length formula, prove that the circumference of a circle of radius r is $2\pi r$.

4. (a) The difference of squares $a^2 - b^2 = (a - b)(a + b)$ is a special case of

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + ab^{n-2} + b^{n-1}),$$

for any $n \geq 2$. Use this formula to find the exact value of $\lim_{x \rightarrow 3} \left[\frac{x^5 - 243}{x - 3} \right]$.

(b) Find a constant k such that the limit $\lim_{t \rightarrow 3} \left[\frac{4t^2 + kt + 7k - 6}{2t^2 - 5t - 3} \right]$ exists.