

31 January 2017

1. Find volumes of the following solids using calculus.

- (a) A right circular cone of radius r and height h .
- (b) A right circular cylinder of height h and radius r .
- (c) A torus of major radius a and minor radius b (you may use geometry for the last step).
- (d) The area between $x = 1$, $x = a > 1$, $y = 0$, and $y = 1/x$ rotated around the x -axis (this shape is known as *Gabriel's horn*).
- (e) A bead of h and width $2r$ (a solid sphere of radius r with a hole drilled through).

2. Recall the product rule and the fact that $\int f'(x) dx = f(x)$ (we omit the constant for now).

- (a) Use these facts to prove the integration by parts formula:

$$\int f(x)g'(x) dx = f(x)g(x) - \int f'(x)g(x) dx.$$

- (b) Use these facts to prove another version of the integration by parts formula:

$$\int f(x)g(x) dx = f(x) \int g(x) dx - \int f'(x) \int g(x) dx dx.$$

3. Evaluate the following integrals. Be careful in choosing which method to use.

(a) $\int x \ln(x) dx$

(e) $\int \frac{x^2 - \sqrt{x}}{2x} dx$

(b) $\int e^{4x} \cos(3x) dx$

(f) $\int (\ln(x))^2 dx$

(c) $\int x^2 \sin(x) dx$

(g) $\int \tan^2(x) dx$

(d) $\int x \sin(3x + 4) dx$

(h) $\int x\sqrt{x+2} dx$