ESP Math 182

Worksheet 2

Spring 2017

12 January 2017

---

1. Suppose f is a continuous, 2π-periodic function with $\int_0^{4\pi} f(t)\ dt = 7$. For any integer k, compute $\int_{k\pi}^{(k+2)\pi} f(t)\ dt$. Hint: 2π-periodicity implies $\int_a^b f(t)\ dt = \int_{b+n2\pi}^{a+n2\pi} f(t)\ dt$ for any integer n.

Recall the fundamental theorems of calculus. Both assume that f is continuous on $[a, b]$

1st FTC: $\int_a^b f(t)\ dt = F(b) - F(a)$ for any antiderivative F of f.

2nd FTC: $\frac{d}{dx}\int_a^x f(t)\ dt = f(x)$ for any $x \in (a, b]$.

Use these to answer the questions below.

2. (a) Show that for positive numbers a and b, $\int_1^a \frac{1}{t}\ dt + \int_1^b \frac{1}{t}\ dt = \int_1^{ab} \frac{1}{t}\ dt$.

   (b) Let $g(x) = \int_a^x f(t)\ dt$. Using g, compute $\frac{d}{dx}\int_{\alpha(x)}^{\beta(x)} f(t)\ dt$ for any functions α, β.

   (c) Show that if $\int_0^x f(t)\ dt = x \cdot f(x)$. then f is a constant function.

3. (a) Describe, in your own words, what is an even function and what is an odd function.

   (b) Do functions that are neither even nor odd exist? If no, why? If yes, give an example.

   (c) Are the two expressions below the same or not? Why?

$$\int_{-1}^{1} \frac{1}{x^2}\, dx \qquad\qquad 2\int_{0}^{1} \frac{1}{x^2}\, dx$$

   (d) Give 3 antiderivatives of the function $f(x) = \frac{1}{x^4}$.

4. Answer the following True / False questions. If True, justify. If False, give a counterexample.

   (a) True or false: A function has a unique antiderivative.

   (b) True or false: An even function cannot be the antiderivative of an odd function.

   (c) True or false: Even functions always have odd functions as antiderivatives.

5. **Bonus:** Find a function $f(t)$ and a number a such that $6 + \displaystyle\int_a^x \frac{f(t)}{t^2}\, dt = 2\sqrt{x}$.