ESP Math 182

Worksheet 7

Spring 2015

3 February 2015

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1. A mineral deposit along a strip of length 6 cm has density $\rho(x) = 0.01x(6 - x)$ g/cm for $0 \leqslant x \leqslant 6$. Calculate the total mass of the deposit.

2. Odzala National Park in the Congo has a high density of gorillas. Suppose that the radial population density is $\rho(r) = 52(1 + r^2)^{-2}$ gorillas per square kilometer, where r is the distance from a large grassy clearing with a source of food and water. Calculate the number of gorillas within a 5km radius of the clearing.

3. Find the average value of the given function for the interval.

   (a) $f(s) = \dfrac{1}{s^2}$, $s \in [2, 5]$

   (b) $g(x) = \dfrac{1}{x^2 + 9}$, $x \in [-1, 1]$

   (c) $k(t) = \sec^2(t)$, $t \in [0, \frac{\pi}{4}]$

4. Assume the prssure P and volume V of a gas in a cylinder of length 30in and radius 3in, with a movable piston, are related by $PV^{1.4} = k$, where k is a constant. When the cylinder is full, the gas pressure is 200 lb/in^2.

   (a) Caluclate k.

   (b) Calculate the force on the piston as a function of the length of the column of gas (force is PA where A is the piston's area).

   (c) Calculate the work required to compress the gas column from 30 to 20 inches.

5. A bead is made by taking a sphere and drilling a hole through the center. Find the volume V of the bead with radius R and where the radius of the hole drilled out is r. Show that $V = \frac{\pi}{6}h^3$, where h is the height of the bead after being drilled. This formula has a surprising consequence: Since V can be expressed in terms of h alone, it follows that two beads of height 2 in., one formed from a sphere the size of an orange and the other the size of the earth would have the same volume! Can you explain intuitively how this is possible?