Worksheet 6

29 January 2015

1. Solve the following word problems.

   (a) A hollow cone is standing on its base. The base has a radius of 3 feet and the cone's height is 10 feet. What volume of water would be needed to fill the cone to a height of 6 feet?

   (b) A hemispherical tank is lying on its flat side. It has a radius of 2 meters. How much water is need to fill the tank to a height of 1.5 meters?

2. (a) Explain, in your own words, the shell method of integration.

   (b) Explain, in your own words, the disk / washer method of integration.

   (c) Let S be the region bounded by $y = \frac{1}{x+1}$ and $y = 1 - \frac{x}{3}$, and let V be the solid of revolution of S about the x-axis.

      i. Use the shell method to determine V.
      ii. Use the disk / washer method to determine V.

3. Solve the following area and volume problems.

   (a) Set up, but do not evaluate the integral expressing the area between the circles $x^2 + y^2 = 2$ and $x^2 + (y+1)^2 = 1$.

   (b) Using integrals, find the vloume of a pyramid of height h whose base is an equilateral triangle of side length s.

4. Find volumes of the following solids using calculus.

   (a) A ball of radius r.

   (b) A right circular cone of radius r and height h.

   (c) A right circular cylinder of height h and radius r.

   (d) The solid obtained by rotating the region enclosed by

   $$y = 16 - x \ , \quad y = 12 - x \ , \quad x = 0$$

   about the line $y = 15$.

5. Find a function $f(t)$ and a number a such that for all $x \geqslant 0$,

   $$6 + \int_a^x \frac{f(t)}{t^2} dt = 2\sqrt{x}.$$