ESP Math 182 **Spring 2015**

Worksheet 5

27 January 2015

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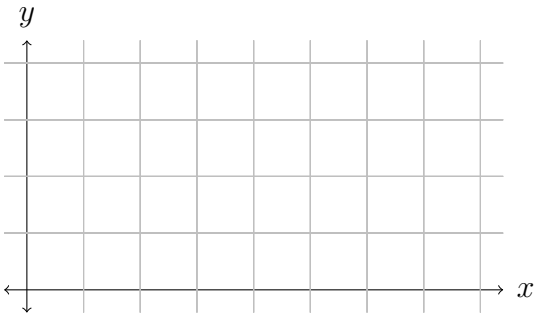
1. Let $f(x) = \sqrt{x}$, $g(x) = x^2$ and $h(x) = 2x$ for $0 \leq x \leq 1$.

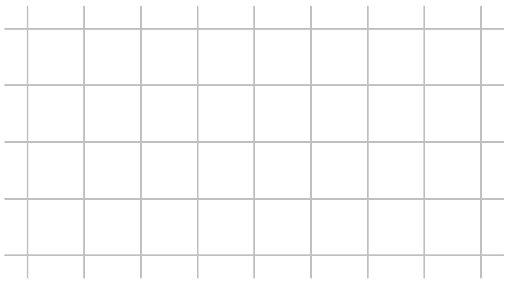
   (a) Draw the graphs of the functions on the given interval on the grid below.



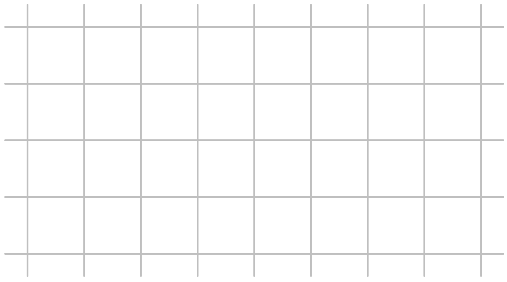
   (b) Find the area of the region with all three of these functions as its boundary on this interval.

2. The following integrals represent the area of some geometric shape. For each, decide what shape the area is representing and give the dimensions of the shape (base, height, radius, etc, as applicable). Make a sketch with labels.

   (a) $\displaystyle\int_0^1 3x \, dx$



   (b) $\displaystyle\int_0^{\sqrt{15}} \sqrt{15 - h^2} \, dh$



3. Find the derivatives $\frac{dy}{dx}$ of the following functions.

   (a) $\tan^{-1}(xy) = x^2 + y^2$

   (b) $\sin(e^y) = (\sqrt{y})^x$

   (c) $y = \left(\cos^{-1}(\sqrt{x})\right)^2$

   (d) $y = \dfrac{\cos^{11}(3x)\ \sqrt[13]{x^2+1}}{e^{\sqrt{x}}\csc^{17}(x)}$

4. Some limit problems.

   (a) The difference of squares $a^2 - b^2 = (a-b)(a+b)$ is a special case of the reduction formula

   $$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + ab^{n-2} + b^{n-1}).$$

   Use this formula to find the exact value of $\lim_{x\to 3}\left[\dfrac{x^5 - 243}{x-3}\right]$.

   (b) Find a constant k such that the limit $\lim_{t\to 3}\left[\dfrac{4t^2 + kt + 7k - 6}{2t^2 - 5t - 3}\right]$ exists.

5. Tricky questions if you have the time (courtesy of M.Rammaha at `math.unl.edu/~mrammaha1`).

   (a) Prove that $e^\pi > \pi^e$. (Hint: Take the natural logarithm of both sides and view the statement as two functions compared at a certain value)

   (b) Suppose that f is a continuous function on $[0,2]$ such that $f(0) = f(2)$. Show that there is a real number $z \in [1,2]$ such that $f(z) = f(z-1)$.