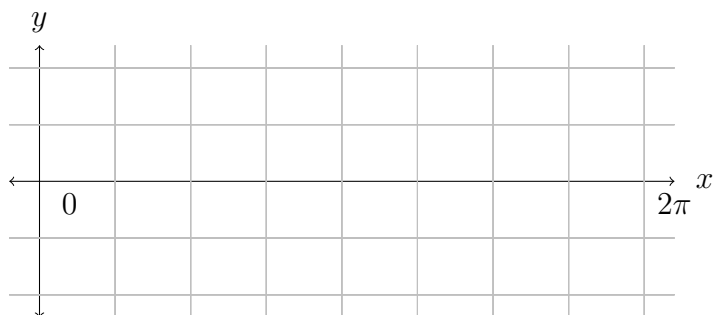


22 January 2015

1. (a) Draw the graphs of the functions $y = \sin \theta$ and $y = \sin 2\theta$ on the grid below, on the interval $[0, 2\pi]$.



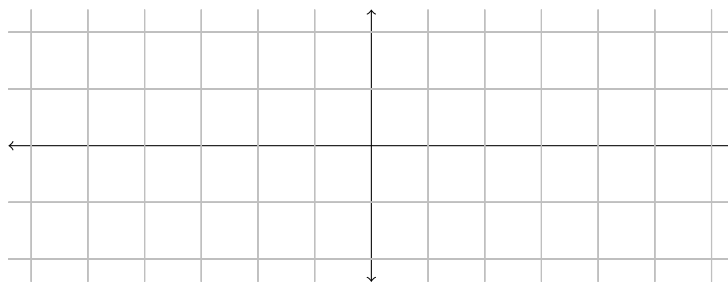
- (b) Find the area between the two curves on the interval $[0, \pi]$.
- (c) How does the area found above compare to the area between the same curves on the interval $[0, 2\pi]$?
- (d) Compute $\int \cos(x) \sin^n(x) dx$, where n is any positive integer.
2. (a) Solve the following derivatives.

$$\frac{d}{dx} \arcsin(x)$$

$$\frac{d}{dx} \arccos(x)$$

$$\frac{d}{dx} \arctan(x)$$

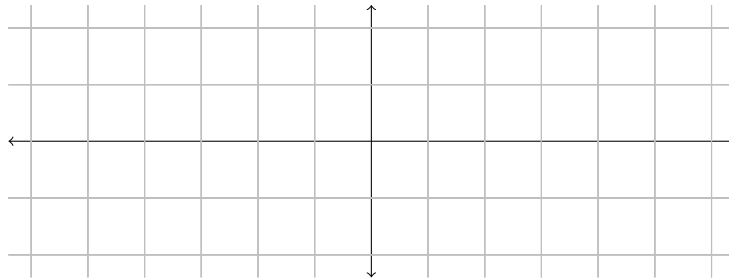
- (b) Sketch the region bounded by the curves $y = \frac{1}{\sqrt{1-x^2}}$ and $y = -\frac{1}{\sqrt{1-x^2}}$ and the lines $x = 1/2$ and $x = -1/2$ on the grid below. Make sure to label the axes appropriately.



- (c) Find the exact value of this area.

3. (a) Where do the curves $x = y^2 + 4y - 22$ and $y = \frac{1}{3}x - \frac{8}{3}$ meet?

(b) Draw the curves $x = y^2 + 4y - 22$ and $y = \frac{1}{3}x - \frac{8}{3}$ on the grid below, so that all intersection points found above are included. Make sure to label the axes appropriately.



(c) Find the area of the region that lies to the right of the parabola and to the left of the line.

4. (a) Find $H'(2)$ given that $H(x) = \int_0^{x^2-4} \frac{1}{1+\sqrt{t}} dt$.

(b) Find $H'(2)$ given that $H(x) = \int_{2x}^{x^3-4} \frac{1}{1+\sqrt{t}} dt$.

(c) Find $H'(3)$ given that $H(x) = \frac{1}{x} \int_3^x 2t - 3H'(t) dt$.

(d) Find $F'(x)$ if $F(x) = \int_0^x xf(t) dt$.

(e) Find all continuous functions $f(x)$ satisfying $\int_0^x f(t) dt = (f(x))^2 + C$. (Hint: Differentiate both sides with respect to x .)