

20 January 2015

1. Find what is wrong with this work. Can you complete it correctly?

$$\int \cos \sqrt{x} \, dx$$

$$\text{Let } u = \sqrt{x}.$$

$$\text{Then } du = \frac{dx}{2\sqrt{x}}.$$

$$\text{Then } 2\sqrt{x} \, du = dx.$$

$$\text{So the integral is } \int 2\sqrt{x} \cos u \, du$$

$$= \frac{4}{3}x^{3/2} \sin u + C = \frac{4}{3}x^{3/2} \sin \sqrt{x} + C.$$

2. Solve these problems by integration by substitution.

(a) $\int \frac{x}{\sqrt{x^2 + 9}} \, dx$

(b) $\int x^2 \sin(x^3) \, dx$

(c) $\int \sin^5(x) \cos(x) \, dx$

(d) $\int \frac{\tan(\ln(x))}{x} \, dx$

(e) $\int (x^7 + 2)(x^8 + 16x - 5)^4 \, dx$

(f) $\int \frac{2x - 1}{x^2 - x} \, dx$

(g) $\int \frac{x^2 e^{\sqrt{x^3 - 3}}}{\sqrt{x^3 - 3}} \, dx$

3. Let $f(x) = 4 - x^2$ and $g(x) - x - 2 = 0$.

(a) What is the definition of a graph?

(b) Where do the graphs of f and g meet?

(c) Given the number of points at which the graphs meet, do they bound a finite area? If so, where is this area?

(d) Find the area of the region bounded by the curves f and g .

4. Let $n \geq 1$.

(a) Draw the graph of $y = x^n$ on $x \in [0, 1]$ for $n = 1, 2, 3$ below.

(b) Find the integral of $y = x^n$ on $x \in [0, 1]$ for any $n \geq 1$.

(c) What happens to your answer when $n \rightarrow \infty$? Does this make sense?