ESP Math 182 | # Worksheet 29 | **Spring 2015**

28 April 2015

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1. *Volumes of revolution:* Calculate the following volumes using the shell method.

   (a) The area in the fourth quadrant bounded by $y = x - 3$ around the x-axis.

   (b) The area in the first quadrant bounded by $y = 1 - x^2$ and $y = x^2$ around the axis $x = 3$.

   (c) The area bounded by $y = \sin(x)$, $y = 0$, $y = \pi$ and $x = 0$ around the axis $y = -2$.

   Calculate the following volumes using the disk method.

   (d) The area bounded by $y = \ln(x)$, $y = -\ln(x) + 1$, $x = \frac{1}{2}$, and $x = 2$ around the x-axis.

   (e) The area in the second quadrant bounded by $x = -y^2$ and $y = x^2$ around the axis $x = -3$.

   (f) The volume of revolution of $y = x(x-1)(x-2)$ revolved around the x-axis between $x = 0$ and $x = 3$.

2. *Series expansions:* Give the Taylor (T) or Maclaurin (M) series of the following functions.

   (a) (T) $\frac{1}{x}$

   (b) (T) $\frac{2}{x}$

   (c) (T) $\ln(x)$

   (d) (T) e^x

   (e) (M) $\sin(x)$

   (f) (M) $\cos(x)$

   (g) (T) x

   (h) (T) x^n

   Using the above, derivatives, integrals, and the sum of a geometric series, give the Maclaurin series of the following functions. Remember to give it in the form $\sum_{n=0}^{\infty} a_n x^n$.

   (i) $\dfrac{1}{(1-x)^2}$

   (j) $3\cos(3x^3)$

   (k) $\dfrac{1}{x^2}$

   (l) $\dfrac{x}{1-x}$

   (m) $(2 + x - x^2)e^x$

   (n) $\dfrac{1}{x-1} + \dfrac{2}{(x-1)^3}$

   Recover the function from the given Maclaurin series.

   (o) $\displaystyle\sum_{n=1}^{\infty} \frac{(2x)^{n-1}}{(n-1)!}$

   (p) $\displaystyle\sum_{n=0}^{\infty} \frac{2(-x^2)^{n+1}}{(2n+1)!}$

3. *Parametric and polar equations:* Convert the following functions from parametric form to an explicit form with y, x or vice versa, as appropriate. Give the interval where the resulting function is defined.

(a) $x = 3t - 5$, $y = t^2 + 5t - 1$

(b) $x = 4\cos(t)$, $y = 3\cos^2(t) + \cos(t)\sin(t) + 3\sin^2(t)$

(c) $y = e^x$

(d) $y = \frac{1}{x} + \frac{2}{x^2}$

(e) $x^2 + y^2 = 4$

Convert the following equations from polar to cartesian or vice versa, as appropriate.

(f) $3x^2 + 3y^2 = 10x$

(g) $x^2 + 7x + 5 + y^2 - 3y + 2 = 10$

(h) $r^2 = r^2\sin^2(\theta) - 2\sin(\theta)$

(i) $5r = \frac{1}{2}\cos(\theta)$

4. *Trigonometric substitution:* Solve the following integrals.

(a) $\displaystyle\int_{2/3}^{4/5} \frac{\sqrt{25x^2 - 4}}{x}\, dx$

(b) $\displaystyle\int \frac{1}{x^4\sqrt{9 - x^2}}\, dx$

(c) $\displaystyle\int \frac{1}{\sqrt{2x^2 - 4x - 7}}\, dx$

(d) $\displaystyle\int e^{4x}\sqrt{1 + e^{2x}}\, dx$