ESP Math 182 **Spring 2015**

W.29 Solutions

28 April 2015

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1. *Volumes of revolution:* Calculate the following volumes using the shell method.

   (a) $\int_0^{-3} 2\pi y(y+3)dy$

   (b) $\int_{1/\sqrt{2}}^{0} 2\pi(3-x)((1-x^2)-x^2)dx$

   (c) $2\int_0^1 2\pi(2+y)\left(\frac{\pi}{2}-\sin^{-1}(y)\right)dy$

   Calculate the following volumes using the disk method.

   (d) $\int_1^{\sqrt{e}} \pi\ln^2(x)dx + \int_{\sqrt{e}}^{2} \pi(-\ln(x)+1)^2dx$

   (e) $\int_0^1 \pi\left((3-y^2)^2-(3-\sqrt{y})^2\right)dy$

   (f) $\int_0^3 \pi(x(x-1)(x-2))^2dx$

2. *Series expansions:* Give the Taylor (T) or Maclaurin (M) series of the following functions.

   (a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{a^{n+1}}(x-a)^n$

   (b) $\sum_{n=0}^{\infty} \frac{2(-1)^n}{a^{n+1}}(x-a)^n$

   (c) $\ln(a)+\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{na^n}(x-a)^n$

   (d) $\sum_{n=0}^{\infty} \frac{e^a}{n!}(x-a)^n$

   (e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1}$

   (f) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}x^{2n}$

   (g) x

   (h) x^n

   Using the above, derivatives, integrals, and the sum of a geometric series, give the Maclaurin series of the following functions. Remember to give it in the form $\sum_{n=0}^{\infty} a_nx^n$.

   (i) $\sum_{n=0}^{\infty} -(n+1)x^n$

   (j) $\sum_{n=0}^{\infty} \frac{3^{2n+1}(-1)^nx^{6n}}{(2n)!}$

   (k) $\sum_{n=0}^{\infty}(n+1)(x+1)^n$

   (l) $\sum_{n=0}^{\infty} x^{n+1}$

   (m) $\sum_{n=0}^{\infty} \frac{(2+2n-n^2)x^n}{n!}$

(n) $\sum_{n=0}^{\infty}(2n^2+6n+7)x^n$

Recover the function from the given Maclaurin series.

(o) e^{2x}

(p) $-2x\sin(x)$

3. *Parametric and polar equations:* Convert the following functions from parametric form to an explicit form with y, x or vice versa, as appropriate. Give the interval where the resulting function is defined.

(a) $\frac{1}{9}x^2+\frac{25}{9}x+\frac{91}{9}$

(b) $\frac{3x\sqrt{16-x^2}}{16}$

(c) $\begin{aligned} x &= t \\ y &= e^t \end{aligned}$

(d) $\begin{aligned} x &= 4t \\ y &= \tfrac{1}{4t}+\tfrac{2}{16t^2} \end{aligned}$

(e) $\begin{aligned} x &= 2\cos(t) \\ y &= \sin(t) \end{aligned}$

Convert the following equations from polar to cartesian or vice versa, as appropriate.

(f) $3r^2 = 10r\cos(\theta)$

(g) $r^2 = 3+3r\sin(\theta)-7r\cos(\theta)$

(h) $x^2\sqrt{x^2+y^2}+2y=0$

(i) $5x^2+5y^2=\frac{1}{2}x$

4. *Trigonometric substitution:* Solve the following integrals.

(a) $\int_{2/3}^{4/5}\frac{\sqrt{25x^2-4}}{x}\,dx$

(b) $\int\frac{1}{x^4\sqrt{9-x^2}}\,dx$

(c) $\int\frac{1}{\sqrt{2x^2-4x-7}}\,dx$

(d) $\int e^{4x}\sqrt{1+e^{2x}}\,dx$