ESP Math 182

Worksheet 26

Spring 2015

16 April 2015

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1. **Warm up:** Find and correct what is wrong with the following statements.

   (a) The divergence test tells if the sum of a series is a number greater than zero.

   (b) Partial fractions works for any fraction of two polynomials.

   (c) The integral of $\arctan(x) = \tan^{-1}(x)$ is $\dfrac{1}{1+x^2}$.

   (d) An improper integral is a type of indefinite integral.

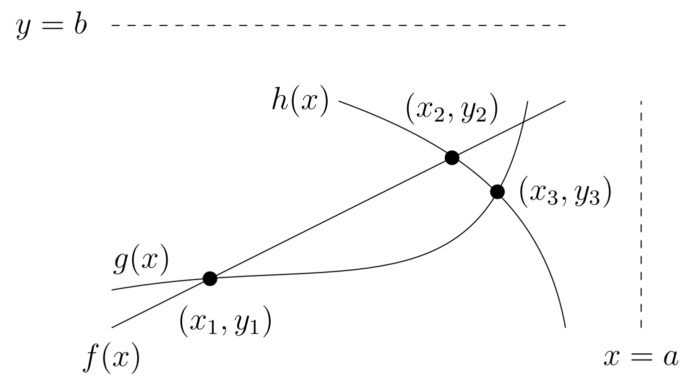
2. Recall the Taylor series expansion for $f(x)$ centered at a, given by $\displaystyle\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$.

   (a) Give the first three terms for the Taylor expansion of $x\ln(x)$ around $a = 1$, and use it to estimate $\ln(4)$.

   (b) Estimate $\ln(4)$ with the first four terms of the Taylor expansion.

   (c) Which is a better approximation of the actual value of $\ln(4)$?

3. Consider the diagram below with three functions $f(x), g(x), h(x)$. Assume that everything in the diagram is contained in the second quadrant. You may also assume the inverse functions $f^{-1}(y)$, $g^{-1}(y)$, $h^{-1}(y)$ are defined everywhere.



(a) *Shell method*: What is the surface area of a cylinder of radius r and height h?

(b) *Washer method*: What is the surface area of a washer with inner radius r_1 and outer radius r_2?

Consider the volume of revolution of the area enclosed by the three curves rotated around $x = a$, using shells.

(c) Which of the following types of integrals will give the volume? Why?

$$\int_{\square}^{\square} \boxed{\qquad\qquad} \, dx \qquad\qquad \int_{\square}^{\square} \boxed{\qquad\qquad} \, dy$$

(d) Which of the following types of expressions will go in the argument of the integral above (the long box in the middle)? Why?

$$2\pi r h \qquad\qquad \pi r^2$$

(e) Give the appropriate values of r and h for the given volume of revolution.

(f) Write the complete expression giving the described volume of revolution.

(g) Write the complete expression giving the volume of revolution for the same area on the diagram, this time:
· rotated around $y = b$, using washers,
· rotated around $x = a$, using washers,
· rotated around $y = b$, using shells.