ESP Math 182

Worksheet 25

Spring 2015

14 April 2015

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1. **Warm up part 1:** Say which of the following series converge and which diverge.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$

   (d) $\displaystyle\sum_{n=1}^{\infty} n$

   (e) $\displaystyle\sum_{n=1}^{\infty} n^n$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n}$

   (g) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^{-n}}$

   (h) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(0.3)^n}$

2. **Warm up part 2:** Consider the following series. Find another series that behaves like the given series for large n. Give your impressions on whether it should converge or diverge.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(n+2)!}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n^2+n-1}}{2n^3-2n+4}$

   (c) $\displaystyle\sum_{n=0}^{\infty} \ln(2^{2n^2+3})(n+2)^{-3}$

3. Recall the geometric series $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ for $|r| < 1$.

   (a) Show that $\frac{1}{1-x} = \frac{-1/3}{1+(x-4)/3}$.

   (b) Using the above, for $|x-4| < 3$, show that $\frac{1}{1-x} = \sum_{n=0}^{\infty} (-1)^{n-1} \frac{(x-4)^n}{3^{n+1}}$.

   (c) For $|x-4| > 3$, show that $\frac{1}{1-x} = \sum_{n=0}^{\infty} (-1)^{n-1} \frac{3^n}{(x-4)^{n+1}}$.

4. Consider the series $f(x) = \sum_{n=0}^{\infty} \frac{(2x)^k}{k!}$.

   (a) Find the derivative $f'(x)$ of the series and rewrite it in terms of $f(x)$.

   (b) Using part (a) above, give the nth derivative of $f(x)$. Do not simply keep taking derivatives of the series.

   (c) What common function is $f(x)$ equal to?

5. Let $f(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{2^k k!}$.

   (a) Show that $f(x)$ has infinite radius of convergence.

   (b) Show that $f'(x) = xf'(x) + f(x)$.