ESP Math 182

Worksheet 24

Spring 2015

9 April 2015

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1. **Warm up:** Answer the following true / false questions.

   (a) A polynomial is a type of power series.

   (b) Every power series is a polynomial.

   (c) A power series may have an interval of convergence not containing zero.

   (d) The interval of convergence of a power series is always twice the length of the radius of convergence.

2. Find the radius of convergence and the interval of convergence of the following series:

   (a) $\displaystyle\sum_{n=1}^{\infty} nx^n$

   (b) $\displaystyle\sum_{n=1}^{\infty} n^n x^n$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{4^n}(2x-1)^n$

3. Recall the geometric series $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$, for $|r| < 1$, which may be viewed as a power series centered at 0.

   (a) Find a power series representation for the function $\frac{1}{z+1}$ centered at 0. Remember to check what happens when $|z| < 1$ and when $|z| > 1$.

   (b) Find a power series representation for the function $\frac{1}{z-2}$ centered at 0. Remember to check what happens when $|z| < 2$ and when $|z| > 2$.

   (c) Find a power series representation for the function $\frac{1}{z^2-z-2}$ centered at 0. You should have three answers, depending on the magnitude of z.