

7 April 2015

1. **Warm up:** Are the following expressions power series? Why or why not?

(a) $\sum_{n=1}^{55} \frac{1}{n}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n}$

(c) $\sum_{n=1}^{\infty} \frac{x^n}{n}$

(d) $\sum_{n=1}^{\infty} \sum_{k=1}^n \frac{x^n}{n^k}$

(e) $\sum_{n=0}^{\infty} x^n n^x$

(f) $\{1, 2, 4, 8, 16, \dots\}$

(g) $\{x, x^2, x^4, x^8, x^{16}, \dots\}$

(h) $1 + 2 + 4 + 8 + 16 + \dots$

(i) $1 + x^2 + x^4 + x^8 + x^{16} + \dots$

(j) π

(k) e^x

Which of the above are non-trivial power series?

2. Show that $\sum_{n=1}^{\infty} n^k 3^{-n}$ converges for all real numbers k .

3. Show that $\sum_{n=1}^{\infty} 2^n x^n$ converges only if $|x| < \frac{1}{2}$.

4. Show that $\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$ diverges by the limit comparison test.

5. Show that $\sum_{n=1}^{\infty} \tan^2\left(\frac{1}{n}\right)$ converges by the limit comparison test.

6. Recall the geometric series $\sum_{n=0}^{\infty} ar^n$.

(a) Write the power series for $\frac{1}{1-x}$ for $|x| < 1$.

(b) What happens when $|x| > 1$? Find a power series of $\frac{1}{1-x}$ for such x .

(c) Calculate the power series for $\frac{1+x}{1-x}$.