ESP Math 182

Worksheet 22

Spring 2015

2 April 2015

Alternating series: An alternating series is a sum $\pm \sum_{n=1}^{\infty} (-1)^n a_n$ with a_n 0 for all n.

Alternating series test: Let $\{a_n\}_{n=1}^{\infty}$ be a sequence that satisfies the following conditions for all n:

1. $a_n \geqslant 0$,

2. $a_{n+1} \leqslant a_n$,

3. $\lim_{n\to\infty} a_n = 0$.

Then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

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1. **Warm up:** Using the definitions above, answer the following questions:

   (a) Is $\sum_{n=1}^{\infty} \frac{(-1)^n}{(-1)^{n+1}} n^{(-1)^n}$ an alternating series?

   (b) If the conditions of the alternating series test are satisfied, can you conclude that the series $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ converges?

   (c) Does $\sum_{n=1}^{\infty} (-1)^n \cos(2n\pi)$ converge by the alternating series test?

   (d) Does $\sum_{n=1}^{\infty} \frac{(-1)^n \cos(2n\pi)}{n}$ converge by the alternating series test?

2. Determine the values of p for which the series $\sum_{n=1}^{\infty} \frac{1}{n \ln(n)^p}$ converges.

3. (a) Show that $\sum_{n=2}^{\infty} \left(\frac{e}{n}\right)^n$ converges.

(b) Show that $\int_1^{\infty} \frac{e^y}{y^y} dy$ converges.

(c) Determine whether or not $\sum_{n=3}^{\infty} \frac{1}{(\ln(n))^{\ln(n)}}$ converges.

4. You are given that for all $x > 0$, the inequalities $x - \frac{x^2}{x} < \ln(1+x) < x$ hold. Use them to answer the following questions.

(a) Prove that $\lim_{x \to 0^+} \left[\frac{\ln(1+x)}{x}\right] = 1$.

(b) Using part (a) above, prove that $\lim_{n \to \infty} \left[\left(1 + \frac{1}{n}\right)^n\right] = e$.

(c) Show that $\sum_{n=1}^{\infty} \frac{a^n n!}{n^n}$ converges for $0 < a < e$ and diverges for $a > e$.

(d) Does the series above converge if $a = e$?