ESP Math 182 **Spring 2015**

Worksheet 21

31 March 2015

Root test: Let a_n be a sequence with $L = \lim_{n\to\infty} \left[\sqrt[n]{|a_n|} \right]$. Then

1. if $0 < L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges;

2. if $L > 1$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

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1. **Warm up:** Let $\sum_{n=1}^{\infty} a_n$ be a series. Give examples of a_n such that

   (a) the series diverges by the divergence test;

   (b) the series converges by the p-test;

   (c) the series converges by the ratio test;

   (d) the root test is inconclusive.

2. Determine if the following series converge or diverge.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{99^n n^3}{100^n}$

   (b) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k k^k}{3^{k^3+1}}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+4n-11}{5n^3-3n^2+2n+4}$

   (d) $\displaystyle\sum_{k=1}^{\infty} \frac{\tan^{-1}(3k)}{\sqrt{2k^3+1}}$

   (e) $\displaystyle\sum_{k=1}^{\infty} \frac{(2k)!}{5^k(k+1)!k!}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[3]{2n^2+1}}{\sqrt[4]{3n^3+2}}$