ESP Math 182

Worksheet 20

Spring 2015

19 March 2015

Divergence test: If $\{a_n\}$ is any sequence such that $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty}[a_n] = 0$.

Limit comparison test: Suppose $\{a_n\}$ and $\{b_n\}$ are sequences such that $a_n \geqslant 0$ and $b_n > 0$ for all n. For $\lim_{n\to\infty}\left[\frac{a_n}{b_n}\right] = L$,

1. if $L \in (0, \infty)$, then $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{n=1}^{\infty} b_n$ converges;

2. if $L = 0$, then if $\sum_{n=1}^{\infty} b_n$ converges, then so does $\sum_{n=1}^{\infty} a_n$;

3. if $L = \infty$, then if $\sum_{n=1}^{\infty} b_n$ diverges, then so does $\sum_{n=1}^{\infty} a_n$.

Ratio test: Let $a_n > 0$ and $L = \lim_{n\to\infty}\left[\frac{a_{n+1}}{a_n}\right]$. Then

1. if $0 < L < 1$, then $\sum_{n=1}^{\infty} a_n$ converges;

2. if $L > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges.

P-series test: $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$.

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1. Using the tests above, determine if the following series converge or diverge. Say which convergence test(s) you are using.

(a) $\sum_{n=1}^{\infty} \frac{n+1}{n}$

(b) $\sum_{n=1}^{\infty} \frac{n^2}{n^3+4}$

(c) $\sum_{n=1}^{\infty} \frac{2}{4n^2-1}$

(d) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

(e) $\sum_{n=1}^{\infty} \frac{1}{2+\sqrt{3n}}$

(f) $\sum_{n=1}^{\infty} \frac{1}{2+3^n}$

(g) $\sum_{n=1}^{\infty} \frac{n^2 2^{n+1}}{3^n}$

(h) $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^3}$

2. (a) Show that $\sum_{n=2}^{\infty}\left(\frac{e}{n}\right)^n$ converges.

(b) Show that $\int_1^{\infty}\frac{e^y}{y^y}dy$ converges.

(c) Determine whether or not $\sum_{n=3}^{\infty}\frac{1}{(\ln(n))^{\ln(n)}}$ converges.

3. You are given that for all $x > 0$, the inequalities $x - \frac{x^2}{x} < \ln(1+x) < x$ hold. Use them to answer the following questions.

(a) Prove that $\lim_{x\to 0^+}\left[\frac{\ln(1+x)}{x}\right] = 1$.

(b) Using part (a) above, prove that $\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right] = e$.

(c) Show that $\sum_{n=1}^{\infty}\frac{a^n n!}{n^n}$ converges for $0 < a < e$ and diverges for $a > e$.

(d) Does the series above converge if $a = e$?