

15 January 2015

1. (a) State, in your own words, the second fundamental theorem of calculus (FTC).

- (b) Use the second FTC to show that for positive numbers, a and b ,

$$\int_1^a \frac{1}{t} dt + \int_1^b \frac{1}{t} dt = \int_1^{ab} \frac{1}{t} dt.$$

2. (a) State, in your own words, the first FTC.

- (b) Use the first FTC to show that if

$$\int_0^x f(t) dt = x \cdot f(x),$$

then f is a constant function.

3. (a) Let $f(x)$ be the function given by

$$f(x) = \int_{a(x)}^{b(x)} g(t) dt.$$

What is the derivative of f with respect to x ?

- (b) State, in your own words, the rule of integration by substitution.

Evaluate the following antiderivatives by either guess and check, or substitution.

(c) $\int e^{12x} dx$

(f) $\int \frac{1}{\sqrt{x+1}} dx$

(d) $\int x \cos(x^2) dx$

(g) $\int e^x \sin(e^x) dx$

(e) $\int \frac{x}{x^2+1} dx$

(h) $\int (2x-1)\sqrt{x^2-x} dx$

4. (a) Describe, in your own words, what is an even function and what is an odd function.

(b) Give 3 antiderivatives of the function $f(x) = \frac{1}{x}$.

(c) True or false: An antiderivative and the indefinite integral of a function are the same.

(d) True or false: The antiderivative of a function is unique.

(e) True or false: An even function cannot be the antiderivative of an odd function.

5. Solve these challenging antiderivatives.

(a) $\int \frac{\tan x}{\cos x} dx$

(b) $\int \frac{1}{x\sqrt{\ln x}} dx$

(c) $\int e^{e^t} e^t dx$